However, intersection computation of two planes $\boldsymbol{\rho}_1, \boldsymbol{\rho}_2$ is a little bit more complicated, if the Euclidean space is used. Using the principle of duality we can write a solution directly using the Plücker coordinates. However, using the GA , we can directly write

$$\boldsymbol{p} = \boldsymbol{\rho}_1 \wedge \boldsymbol{\rho}_2 \tag{44}$$

TABLE IV. GEOMETRIC PRODUCT IN PROJECTIVE SPACE - PLANES

$\boldsymbol{\rho}_1\boldsymbol{\rho}_2$		$\boldsymbol{\rho}_2$			
		a_2	b_2	c_2	d_2
$\boldsymbol{\rho}_1$	a_1	a_1a_2	a_1b_2	a_1c_2	a_1d_2
	b_1	b_1a_2	b_1b_2	b_1c_2	b_1d_2
	c_1	c_1a_2	c_1b_2	c_1c_2	b_1d_2
	d_1	d_1a_2	d_1b_2	d_1c_2	d_1d_2

The table above is actually a result of the geometric product of two planes $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$

$$\begin{gathered}\boldsymbol{\rho}_1\boldsymbol{\rho}_2 \triangleq \boldsymbol{\rho}_1\boldsymbol{\rho}_2^T = \boldsymbol{\rho}_1 \otimes \boldsymbol{\rho}_2 = \boldsymbol{Q} \\ = \begin{bmatrix} a_1a_2 & a_1b_2 & a_1c_2 & a_1d_2 \\ b_1a_2 & b_1b_2 & b_1c_2 & y_1d_2 \\ c_1a_2 & c_1b_2 & c_1c_2 & z_1d_2 \\ d_1a_2 & d_1b_2 & d_1c_2 & d_1d_2 \end{bmatrix} \\ = \boldsymbol{B} + \boldsymbol{U} + \boldsymbol{D}\end{gathered} \tag{45}$$

It means that we have computation of the Plücker coordinates for the both cases, i.e. for computation of a line $\boldsymbol{p} = \boldsymbol{\rho}_1 \wedge \boldsymbol{\rho}_2$ or $\boldsymbol{p} = \boldsymbol{x}_1 \wedge \boldsymbol{x}_2$ is given as:

- a union of two points in E^3 and
- an intersection of two planes in E^3

using the projective representation and the principle of duality.

It should be noted that the given approach offers:

- significant simplification of computation of the Plücker coordinates as it is simple and easy to derive and explain,
- uses *vector-vector* operations, which is especially convenient for SSE and GPU application
- one code sequence for the both cases

As the Plücker coordinates are also in mechanical engineering applications, especially in robotics due to its simple displacement and momentum specifications, and in other fields simple explanation and derivation is another very important argument for GA approach application.

VII. CONCLUSION

In this contribution, a new reformulation of the Plücker coordinates is presented based on geometric product. It uses geometric algebra approach and standard linear algebra with projective representation. Application of the principle of duality leads to a simple formulations for the both cases, i.e. for the line given by two points or by two intersection points in E^3 . The proposed approach is convenient for GPU application as well, as formulation is based on *vector-vector* operations.

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