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Appendix A

Computation of a bivector area in the *n*-dimensional space can be made as follows:

$$cos(\alpha_{ij}) = \frac{\mathbf{a}_i.\mathbf{a}_j}{||\mathbf{a}_i||\ ||\mathbf{a}_j||} \qquad sin(\alpha_{ij}) = \frac{||\mathbf{a}_i \wedge \mathbf{a}_j||}{||\mathbf{a}_i||\ ||\mathbf{a}_j||} \tag{1}$$

Therefore the square of the bivector area is given as

$$||\mathbf{a}_i \wedge \mathbf{a}_j||^2 = ||\mathbf{a}_i||^2 ||\mathbf{a}_j||^2 sin^2(\alpha_{ij}) \tag{2}$$

As the following identity is valid

$$sin^2(\alpha_{ij}) = 1 - cos^2(\alpha_{ij}) \tag{3}$$

then it can be expressed as

$$||\mathbf{a}_i \wedge \mathbf{a}_j||^2 = ||\mathbf{a}_i||^2 ||\mathbf{a}_j||^2 (1 - cos^2(\alpha_{ij})) \tag{4}$$

by a substitution of Eq.1

$$||\mathbf{a}_i \wedge \mathbf{a}_j||^2 = ||\mathbf{a}_i||^2 ||\mathbf{a}_j||^2 (1 - \frac{(\mathbf{a}_i \cdot \mathbf{a}_j)^2}{||\mathbf{a}_i||^2\ ||\mathbf{a}_j||^2}) \tag{5}$$

and algebraic manipulation

$$||\mathbf{a}_i \wedge \mathbf{a}_j||^2 = ||\mathbf{a}_i||^2 ||\mathbf{a}_j||^2 (\frac{||\mathbf{a}_i||^2 \cdot ||\mathbf{a}_j||^2 - (\mathbf{a}_i \cdot \mathbf{a}_j)^2}{||\mathbf{a}_i||^2 ||\mathbf{a}_j||^2}) \tag{6}$$

i.e.

$$||\mathbf{a}_i \wedge \mathbf{a}_j||^2 = ||\mathbf{a}_i||^2 ||\mathbf{a}_j||^2 \frac{||\mathbf{a}_i||^2 ||\mathbf{a}_j||^2 - (\mathbf{a}_i \cdot \mathbf{a}_j)^2}{||\mathbf{a}_i||^2 ||\mathbf{a}_j||^2} \tag{7}$$

Now, reduction can be used

$$||\mathbf{a}_i \wedge \mathbf{a}_j||^2 = ||\mathbf{a}_i||^2 ||\mathbf{a}_j||^2 - (\mathbf{a}_i \cdot \mathbf{a}_j)^2 \tag{8}$$

and finally the square of the area of a bivector is given as:

$$||\mathbf{a}_i \wedge \mathbf{a}_j|| = \sqrt{(||\mathbf{a}_i||^2 ||\mathbf{a}_j||^2 - (\mathbf{a}_i \cdot \mathbf{a}_j)^2)} \tag{9}$$