Function number	Radial basis function			
	CS-RBF	TPS	Gauss	Proposed
1	$7.33{\cdot}10^{-2}$	$7.76{\cdot}10^{-3}$	$1.70 \cdot 10^{-3}$	$9.21 \cdot 10^{-4}$
2	$1.81{\cdot}10^{0}$	$5.24{\cdot}10^{-3}$	$5.19 \cdot 10^{-1}$	$6.29 \cdot 10^{-3}$
3	$1.44{\cdot}10^{-1}$	$1.39{\cdot}10^{-2}$	$5.09 \cdot 10^{-2}$	$2.30 \cdot 10^{-3}$
4	$1.33{\cdot}10^{-1}$	$1.52{\cdot}10^{-2}$	$1.76 \cdot 10^{-3}$	$1.89 \cdot 10^{-3}$

TABLE IV: Maximum absolute error.

Function number	Radial basis function			
	CS-RBF	TPS	Gauss	Proposed
1	$6.60{\cdot}10^{-19}$	$8.80{\cdot}10^{-6}$	$1.44 \cdot 10^{-12}$	$2.49 \cdot 10^{-9}$
2	$2.68{\cdot}10^{-18}$	$8.47{\cdot}10^{-5}$	$2.43 \cdot 10^{-12}$	$1.05 \cdot 10^{-7}$
3	$4.05{\cdot}10^{-18}$	$4.58{\cdot}10^{-6}$	$2.35 \cdot 10^{-12}$	$2.25 \cdot 10^{-9}$
4	$2.78{\cdot}10^{-18}$	$2.44{\cdot}10^{-6}$	$9.17 \cdot 10^{-14}$	$3.20 \cdot 10^{-10}$

TABLE V: Condition numbers.

tional points are not included in the approximation. (maximum error for the function 2, if CS-RBF is used).

The proposed RBF approximation method was also tested for a newly developed RBF. The experiments proved significant precision improvement of the final approximation over the TPS function.

VIII. Conclusion

In this contribution a novel approach for RBF approximation based on geometrical properties of a sampled curve (signal) is presented. Experiments proved advantages of the global functions over CS-RBFs are sensitive to the shape parameter selection and require more points for acceptable approximation in general.

The newly developed RBF function is better in the precision terms over the TPS function, however, the TPS function has a little bit conditionality of the RBF metrics.

The experiments also proved that the CS-RBFs require variable shape parameter which is significant result as CS-RBFs are used in many areas, e.g. solution of partial differential equations, etc. The adaptive shape parameter for CS-RBFs is to be explored in future.

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