RADIAL BASIS FUNCTION USE FOR THE RESTORATION OF DAMAGED IMAGES

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Abstract: Radial Basis Function (RBF) can be used for reconstruction of damaged images, filling gaps and for restoring missing data in images. Comparisons with standard method for image inpainting and experimental results are included and demonstrate the feasibility of the use of the RBF method for image processing applications.

Key words: inpainting, radial basis functions, interpolation, image processing

1. INTRODUCTION

One of the interesting problems is how to reconstruct an image well possible from damaged or incomplete original as. This problem is referred to in many papers[1]. The main question is: “What value was in a corrupted position and how can I restore it ?” The Radial Basis Function method (RBF) is based on variational implicit functions principle and can be used for interpolation of scattered data. The possibility of missing data restoration (image inpainting) by the RBF method was mentioned in Kojekine & Savchenko[2]. They used this method for surface retouching and marginally for image inpainting as well. They used compactly supported radial basis functions (CSRBF)[3] for reconstruction and octree data structure for representation of the parts for reconstruction. The advantage of this method is that the linear system is sparse and can be solved easily[4]. The drawback of

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this approach is in error which can be obtained with an improper selection of the radius of support of the CSRBF.

In this paper we used a global radial basis function for image reconstruction, inpainting and drawing removal.

2. PROBLEM DEFINITION

Let us assume that we have an image Ω with resolution *M x N* with 256 gray levels. Some pixels have incorrect values (missing or overwritten), see Fig. 1(a-c). We would like to restore the original image or remove inpainting etc. Let us assume that we can detect "missing pixels", pixels with corrupted values or inpainted pixels[5], too. For our experiments we used original images, see Fig. 1d, and noise, writing or drawing was used to corrupt them.

Note that restoration of the original image is related to scattered data interpolation problem, where many points are not defined and we want to find a value for them.

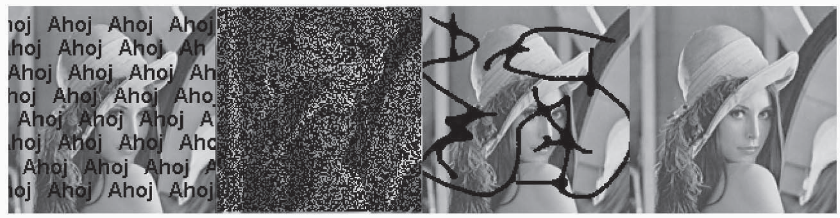

Figure 1. Images with inpainting, noise, scratches and the original one (a, b, c, d).

3. RADIAL BASIS FUNCTIONS

Let us describe the RBF method now. The RBF method may be used to interpolate a smooth function given by *n* points. The resulting interpolating function thus becomes[6]:

$$f(\mathbf{x}) = \sum_{j=1}^{n} \lambda_j \phi(\| \mathbf{x} - \mathbf{c}_j \|) + P(\mathbf{x}), \quad \sum_{i=1}^{n} \lambda_i c^x = \sum_{i=1}^{n} \lambda_i c^y = \sum_{i=1}^{n} \lambda_i = 0 \quad (1, 2)$$

where $f(\mathbf{c}_i)=h_i$, for i=1,…,n, $\mathbf{c}_j$ are given locations of a set of *n* input points (pixels), λ_j are unknown weights, $\mathbf{x}$ is a particular point and $\phi(\|\mathbf{x}-\mathbf{c}_j\|)$ is a radial basis function, $\|\mathbf{x}-\mathbf{c}_j\|=r_j$ is the Euclidean distance (of pixels in our

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case) and $P(\mathbf{x})$ is a polynomial of degree m depending on the choice of ϕ. There are some popular choices for the basis function, e.g. the thin-plate spline $\phi(r) = r^2 * \log(r)$, the Gaussian $\phi(r) = \exp(-\xi r^2)$, the multiquadric $\phi(r) = \sqrt{(r^2+\xi^2)}$, biharmonic $\phi(r) = |r|$ and triharmonic $\phi(r) = |r|^3$ splines, where ξ is a parameter.

Now we have the linear system of equations Eq. (1) with unknowns λ_j, a_x, a_y, a_z. Natural additional constraints for the coefficients λ_j must be included in Eq. (2) to ensure orthogonality of a solution. These equations and constraints determine the linear system:

$$\mathbf{B}\left[\frac{\boldsymbol{\lambda}}{\mathbf{a}}\right] = \left[\frac{\mathbf{h}}{\mathbf{0}}\right], \text{ where } \mathbf{B} = \left[\frac{\mathbf{A} \mid \mathbf{P}}{\mathbf{P}^T \mid \mathbf{0}}\right] \text{ and } \mathbf{P} = \begin{bmatrix} c_1^x & c_1^y & 1 \\ \vdots & \vdots & \vdots \\ c_n^x & c_n^y & 1 \end{bmatrix} \tag{3}$$

$$\mathbf{A}_{i,j} = \phi(\| \mathbf{c}_i - \mathbf{c}_j \|), \quad i, j = 1, \dots, n$$

$$\mathbf{a} = [a_x, a_y, a_z]^T, \boldsymbol{\lambda} = [\lambda_1, \lambda_2, \dots, \lambda_n]^T, \; h = [h_1, h_2, \dots, h_n]^T$$

The polynomial $P(\mathbf{x})$ in Eq. (1) ensures positive-definiteness of the solution, of matrix B[3]. Afterwards, the linear equation system Eq. (3) is solved and the solution vector with $\boldsymbol{\lambda}$ and $\mathbf{a}$ is known, then the function $f(\mathbf{x})$ can be evaluated for an arbitrary point $\mathbf{x}$ (a pixel position in our case)[3,7,8,9].

4. IMAGE RESTORATION

For image reconstruction we used the RBF method mentioned above and applied it within a 5 x 5 window of pixels.

```
LoadImage(Ω);      DefineNeighborhood(5,5);
Repeat
For (i,j=1;i<=M,j<=N;i++,j++) {
  if(Hole(i,j)) { /* pixel [i,j] is not defined */
     K = SelectNeighborhoodOfPixel(i,j);
     DeleteHoles(K);  /* remove all undefined pixels */
  CreateSystemAndSolve(K);
     Pixel[i,j] = ComputeValueFromSystem(i,j);}}
Until (all pixels reconstructed)
```

For this window the RBF function was computed and used iteratively to compute the "missing" pixel values. The RBF function differs from case to case as in the window several pixels might be missing. Now we can specify the proposed algorithm (above).

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If there are too many undefined pixel values in the specified window of the size 5 x 5 pixels, then the undefined pixel is not restored and algorithm continues. The missing pixels are restored in next iteration.

5. RESULTS

The LU factorization method was used for solving the system and $\phi(r) = r^2 * \log(r)$ was used as the RBF function in our experiments.
For evaluation of the proposed method we used a following criterion:

$$S^k = \left(\sum_{i=1}^{M} \sum_{j=1}^{N} \left| (\Omega_1(i,j) - \Omega_2(i,j)) \right|^k \right) \cdot \frac{1}{M \cdot N} \tag{4}$$

where: Ω_1 is the original image (without corruption), Ω_2 is the reconstructed image, k = 1 (linear) or k =2 (quadratic differences). We used several images that were corrupted by noise, text and drawing inpainting and the only "Lena image"[5] example is presented here. The results of the reconstruction of the corrupted image, see Fig. 1(a-c). are presented in Fig. 2. Table 1. presents results obtained for the case when only Q [%] of pixels left from the original image.

Table 1. Results.

Lena	Text (a)	Random gaps (b)	Scratches (c)
Q	19 %	81 %	16 %
S^1	1.64	5.00	2.56
S^2	45.38	120.98	103.17

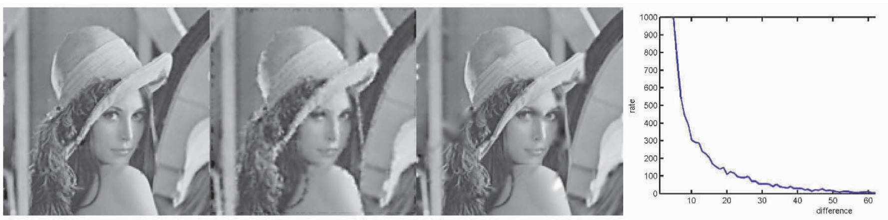

Figure 2. Reconstructed images (from left: text, random gaps, scratches) and histogram with difference between original and reconstructed image (a, b, c, d).

It can be seen, that the results especially for case Fig. 1b. are very good. Also we compared our results with the result of Bertalmio et. al[10]. Original image for reconstruction is presented in Fig. 3a and our result is in Fig. 3b.

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Histogram in Fig. 2d. presents behavior of a difference between original and reconstructed images. It can be seen that the proposed method has a good property.

The quality of the reconstructed image is presented in Fig. 4. on a selected detail Fig. 4a, Fig. 4b presents our results and Fig. 4c the result of Bertalmio[10].

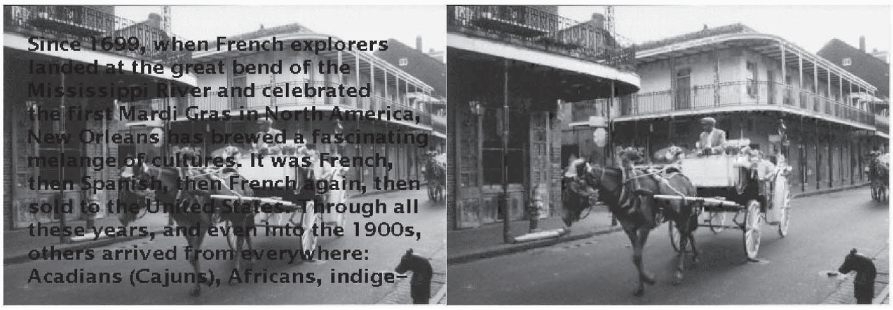

Figure 3. Inpainted image and its reconstruction.

Figure 4. Scaled parts of images.

Figure 5. Present results of Bertholmio (a) and our reconstruction (b).

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