Computer Graphics Forum, Vol.15, No.1, pp.61-68, 1996.

Line Clipping by Convex and Nonconvex Polyhedra in E^3

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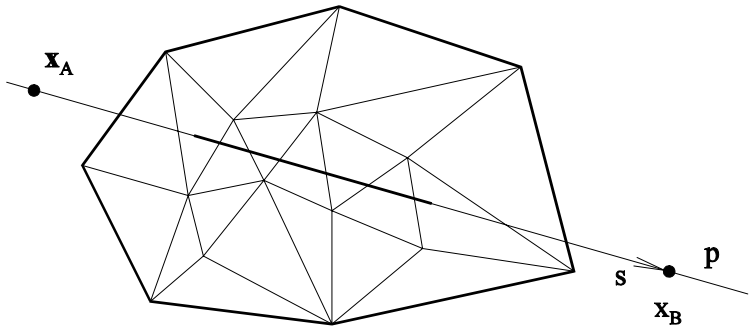
Abstract

A new algorithm for clipping lines against convex polyhedron with $O(N)$ complexity is given with modification for nonconvex polyhedron. The suggested algorithm is faster for higher number of facets of the given polyhedron than the traditional Cyrus-Beck's algorithm. Some principal results of comparisons of all algorithms are shown and give some ideas how the proposed algorithm could be used effectively.

Keywords: Line Clipping, Convex Polyhedron, Non-Convex Polyhedron, Computer Graphics, Algorithm Complexity, Geometric Algorithms, Algorithms Analysis.

1. Introduction

A problem of line clipping against convex polyhedron in E^3 can be solved by Cyrus-Beck's algorithm (CB) [1] for the three dimensional case. Many algorithms for line clipping in E^2 have been published so far with $O(N)$ or $O(\lg N)$ complexities, see [3], [4] for all known references and comparison of algorithms. Nevertheless algorithms for E case are mostly based on the CB algorithm and restricted to convex polyhedron, or based on direct intersection computation of a facet (usually a triangular facet) of the given convex or nonconvex polyhedra and the given line p. Many algorithms for line clipping are restricted to orthogonal or pyramidal volumes, see [2]. Because the line clipping problem solution is a bottleneck for many packages and applications it would be desirable to use the fastest algorithm even it is of complexity $O(N)$.



Line clipping by convex and non-convex polyhedra in E^3

Figure 1

[1] Supported by the grant UWB-156/1995; Published in Computer Graphics Forum Vol.15, No.1,pp.61-68

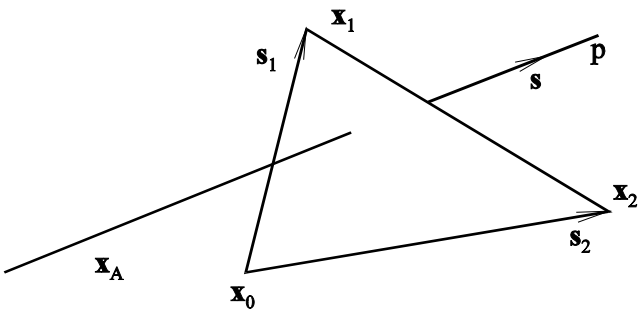
```
procedure Clip_3D_Cyrus_Beck (x_A, x_B);
begin  { !! all vectors n_i are precomputed !! algorithm shortened }
       t_min := 0.0 ;    t_max := 1.0 ;    s := x_A − x_B;
       { for a line initialization:    t_min := −∞ ;    t_min := ∞ ; }
       i := 1;
       while i <= N do { N is a number of facets }
       begin  { n_i is a normal vector of the i-th facet }
              { and n_i must point out of the convex polyhedron }
              { v_j is the j-th vertex of the polyhedron }
              ξ := s^T n_i ;    s_i := x_i − x_A ;
              if ξ ≠ 0.0 then
              begin  t := s_i^T n_i / ξ;
                     if ξ > 0.0 then t_max := min(t, t_max) else t_min := max(t, t_min) ;
              end else { line is parallel to the i-th facet } Special case solution;
              i := i + 1;
       end;
       if t_min > t_max then { the line doesn't intersect the polyhedron }
              EXIT; { !! < t_min, t_max > = ∅ }
       { recompute endpoints of the line segment if changed }
       { for lines points x_A, x_B must be recomputed always }
       if t_max < 1.0 then x_B := x_A + s t_max ;
       if t_min > 0.0 then x_A := x_A + s t_min ;
       SHOW_LINE(x_A, x_B);
end { Clip_3D_Cyrus_Beck };
```

Algorithm 1

The main disadvantage of the CB algorithm is direct line intersection computation for all planes which form the boundary of the given convex polyhedron.

It means that **N-2 of intersection computations are wasted**, if N is a number of facets of the given convex polyhedron., The main advantages of the CB algorithm are its stability and that facets that forms the polyhedron surface might be generally polygons.

The efficiency of the CB algorithm is given by a **simple** algorithm for direct intersection computation of a line with a plane, see alg.1. It is obvious that with growing number of facets of the given polyhedron the efficiency decreases as many invalid intersection points are computed.



Line p intersects the triangular facet

Figure 2

Let us consider only triangular facets of the given polyhedron for the following (generally it is not necessary). For the given line p it is necessary to find an effective test whether the line p intersects the given triangle, see fig.2. The intersection points for convex and nonconvex polyhedra can be directly computed as a solution of parametric equations

$$\mathbf{x}(t) = \mathbf{x}_A + \mathbf{s}\,t \qquad t \in (-\infty,+\infty) \qquad (\alpha)$$

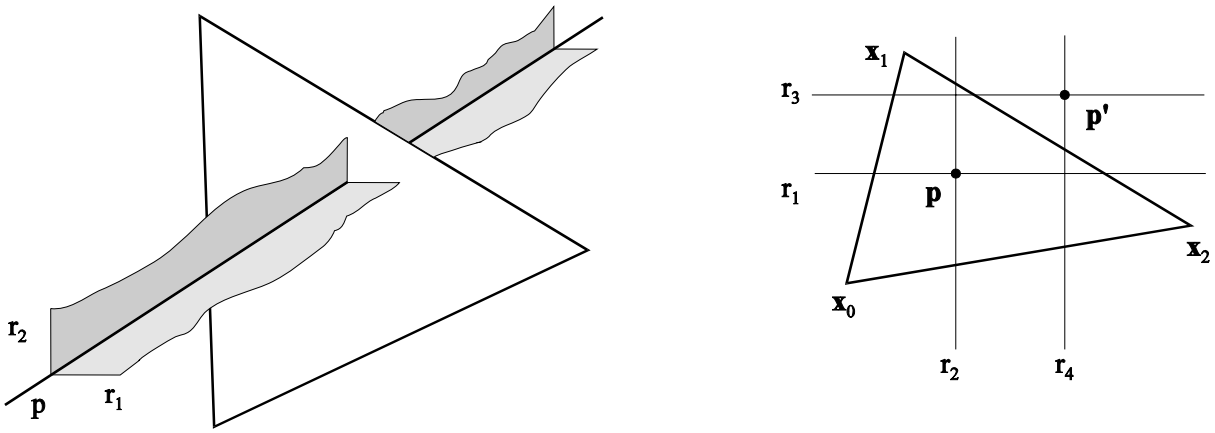
$$\mathbf{x}(p,q) = \mathbf{x}_0 + \mathbf{s}_1\,p + \mathbf{s}_2 q \qquad p,q \in <0,1> \;\&\; p+q \le 1 \qquad (\beta)$$

e.g. in the matrix form

$$\left[\, \mathbf{s}_1 \,|\, \mathbf{s}_2 \,|\, -\mathbf{s} \right] \bullet \begin{bmatrix} p \\ q \\ t \end{bmatrix} = \mathbf{x}_A - \mathbf{x}_0$$

where α is an equation for a given line,
β is an equation for a given triangular facet.

Unfortunately a test which rejects all lines not intersecting a triangular facet is nearly as complex as the direct computation of the intersection point between the extended facet and the line, used in the CB algorithm. In order to substantially speed up line clipping in E^3, we will develop a simple test, which rejects some of the lines not intersecting the facet.



Usage of two planes for line definition

Figure 3

2. Proposed algorithm

A line p can be defined as an intersection of two nonparallel planes ρ_1 and ρ_2 . It can be seen that if the line p intersects the given triangle then planes ρ_1 and ρ_2 intersect the given triangle, too, but **not vice versa** (planes ρ_3 and ρ_4), see fig.3.

It is possible to test all triangles (facets) of the given polyhedron against ρ_1 and ρ_2 planes. If both planes ρ_1 and ρ_2 intersect the given triangle (facet) then **compute detailed intersection** test, see alg.2. The intersection of the given plane ρ_i and the triangle exists **if and only if** two vertices $\mathbf{x}_j$ and $\mathbf{x}_k$ of the triangle exist so that

$$sign(F_i(x_j)) \neq sign(F_i(x_k))$$

where $F_i(\mathbf{x}) = a_i x + b_i y + c_i z + d_i$ is a separation function for the i-th plane ρ_i, i=1,2,
and $F_i(\mathbf{x}) = 0$ is an equation for the i-th plane ρ_i, i=1,2.

Unfortunately there is some principal inefficiency in this implementation of the proposed solution as the separation function $F_1(\mathbf{x})$, resp. $F_2(\mathbf{x})$ are computed many times than needed because **every vertex is shared** by at least three triangles. Therefore it seems to be convenient if the values $sign(F_1({}^i\mathbf{x}_k))$ are precomputed (${}^i\mathbf{x}_k$ is the k-th vertex of the i-th facet) and stored in a separate vector, see alg.3. The separation function $F_2(\mathbf{x})$ is not used for all vertices but only for those that are intersected by the plane ρ_1 and therefore the separation function is used directly without precomputation of the $sign(F_2({}^i\mathbf{x}_k))$ values.

It means that the proposed solution might significantly improve the efficiency, especially for higher N, see alg.3.

```
procedure CLIP_3D (x_A, x_B);
begin  t_min := 0.0 ;      t_max := 1.0 ;
       s := x_A − x_B;
       { for a line initialization: t_min := −∞ ; t_min := ∞ ; }
       { ρ1: a1x + c1z + d1 = 0                ρ2: b2y + c2z + d2 = 0 }
       i := 1;          j := 0;
       while i <= N do { N is a number of facets }
       begin  { ⁱx_k means a k-th vertex of the i-th triangle }
              if sign(F1(ⁱx0)) = sign(F1(ⁱx1)) then
                     if sign(F1(ⁱx0)) = sign(F1(ⁱx2)) then goto 1;
                     { do nothing ρ1 does not intersect the i-th triangle }
              if sign(F2(ⁱx0)) = sign(F2(ⁱx1)) then
                     if sign(F2(ⁱx0)) = sign(F2(ⁱx2)) then goto 1;
                     { do nothing ρ2 does not intersect the i-th triangle }
              { both planes ρ1 , ρ2 intersect the i-th triangle }
              { detailed test SOLVE finds a value t }
              if SOLVE (x_A, s , i , t_{j+1}) then j := j + 1;
                            { otherwise t_{j+1} will be rewritten }
1:            i := i +1;
       end { while };
       if j = 0 then EXIT; {no one triangle intersected by the line }
       if t1 > t2 then SWAP (t1, t2); { swaps values t1, t2 }
       t_max := min(t2, t_max) ;   t_min := max(t1, t_min)
       if t_min ≤ t_max then SHOW_LINE( x(t_min), x(t_max) );
end { CLIP_3D };
```

Algorithm 2

Because a line can intersect **the convex polyhedron only** in two points it is possible to extend the condition in the while statement, see alg.2, by condition **and** (j < 2). It can be seen that the detailed test (SOLVE procedure that uses parametric equations) is invoked only on a relatively few facets.

```
procedure CLIP_3D_MOD (x_A, x_B);
```

```
begin  t_min := 0.0 ;      t_max := 1.0 ;      s := x_A − x_B;
       { for a line initialization: t_min := −∞ ; t_min := ∞ ; }
       { ρ1: a1x + c1z + d1 = 0                ρ2: b2y + c2z + d2 = 0 }
       i := 1;          j := 0;
       for k := 1 to N_V do { N_V number of vertices }
              Q := sign(F1(x_k)); { Q is a vector of int or char types }
       i := 1;          j:=0;
       while (i <= N) and (j < 2) do
       begin  { ix_k means a k-th vertex of the i-th triangle }
              { INDEX(i,k) gives the index of k-th vertex }
              { of the i-th triangle, i.e. ix_k = x_INDEX(i,k) }
              if Q_INDEX(i,0) = Q_INDEX(i,1) then
                     if Q_INDEX(i,0) = Q_INDEX(i,2) then goto 1;
                            { do nothing ρ1 does not intersect the i-th triangle }
              if sign(F2(x_INDEX(i,0))) = sign(F2(x_INDEX(i,1))) then
                     if sign(F2(x_INDEX(i,0))) = sign(F2(x_INDEX(i,2))) then goto 1;
                            { do nothing ρ2 does not intersect the i-th triangle }
              { both planes ρ1, ρ2 intersect the i-th triangle }
              { detailed test SOLVE finds a value t },
              if SOLVE (x_A, s , i , t_j+1) then j := j + 1;  { otherwise t_j+1 will be rewritten }
1:            i := i +1;
       end { while };
{*}    if j = 0 then EXIT; {no intersection }
{*}    if t1 > t2 then SWAP (t1, t2); { swaps values t1, t2 }
{*}    t_max := min(t1, t_max) ;   t_min := max(t2, t_min)
{*}    if t_min ≤ t_max then SHOW_LINE( x(t_min), x(t_max) );
end { of CLIP_3D_MOD };
```

Algorithm 3

Planes ρ_1 and ρ_2 can be taken as diagonal to any selected two co-ordinate axes, see fig.4. If those planes ρ_1 and ρ_2 are observed from the line p they are orthogonal. In that case the functions $F_i(\mathbf{x})$ can be simplified so that

$$F_1(\mathbf{x}) = a_1 x + c_1 z + d_1 \qquad \text{for the plane } \rho_1$$
$$F_2(\mathbf{x}) = b_2 y + c_2 z + d_2 \qquad \text{for the plane } \rho_2$$

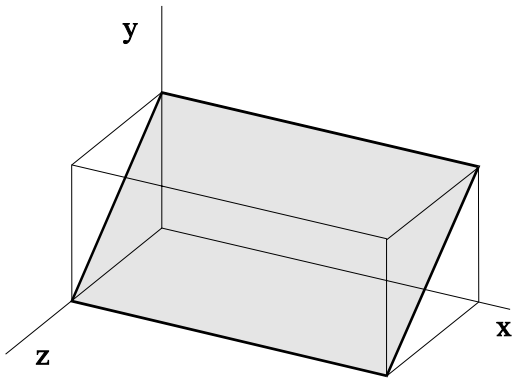
Because planes ρ_1 and ρ_2 are selected as diagonal to the y-axis and x-axis of the co-ordinate system. It can be seen that we get a singular case if the given line is diagonal to the x-axis or y-axis. Therefore we should find a criterion how to select planes ρ_1 and ρ_2 and how to avoid such a singular cases, see fig.4.

Let us suppose that we can generally select two planes ρ_1 and ρ_2, see alg.4, from "diagonal planes" ρ_1' , ρ_2' and ρ_3' which are defined as

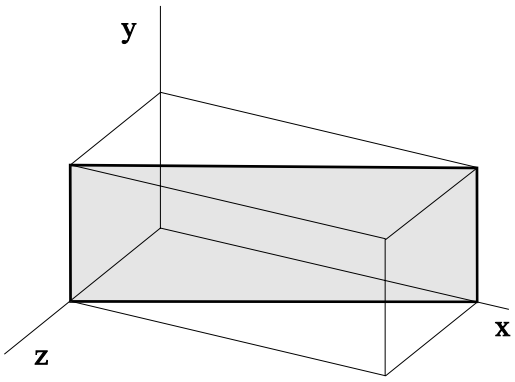
ρ_1': $\mathbf{n}_1^T \mathbf{x} + d_1 = 0$, where $\mathbf{n}_1 = \mathbf{s} \times \mathbf{e}_x$, $\mathbf{e}_x = [1,0,0]^T$

ρ_2': $\mathbf{n}_2^T\mathbf{x}+d_2=0$, where $\mathbf{n}_2=\mathbf{s}\times\mathbf{e}_y$, $\mathbf{e}_y=[0,1,0]^T$

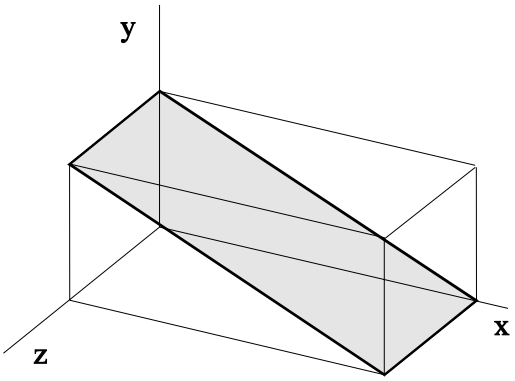
ρ_3': $\mathbf{n}_3^T\mathbf{x}+d_3=0$, where $\mathbf{n}_3=\mathbf{s}\times\mathbf{e}_z$, $\mathbf{e}_z=[0,0,1]^T$



(a) Plane diagonal to x axis



(b) Plane diagonal to y axis



(c) Plane diagonal to z axis

Definition of "diagonal planes"

Figure 4

Planes ρ_1 and ρ_2 are selected so that $\rho_1,\rho_2\in\{\rho_1'\ ,\ \rho_2'\ ,\ \rho_3'\}$

ii := index of **maximal** value $\{|\mathbf{s}_x|\ ,|\mathbf{s}_y|\ ,|\mathbf{s}_z|\}$;

case ii **of**

1: **begin** $\mathbf{n}_1=\mathbf{s}\times\mathbf{e}_y$; $d_1=-\mathbf{n}_1^T\mathbf{x}_A$;
$\mathbf{n}_2=\mathbf{s}\times\mathbf{e}_z$; $d_2=-\mathbf{n}_2^T\mathbf{x}_A$; { see fig.4.a },
end;

2: **begin** $\mathbf{n}_1=\mathbf{s}\times\mathbf{e}_z$; $d_1=-\mathbf{n}_1^T\mathbf{x}_A$;
$\mathbf{n}_2=\mathbf{s}\times\mathbf{e}_x$; $d_2=-\mathbf{n}_2^T\mathbf{x}_A$; { see fig.4.b },
end;

3: **begin** $\mathbf{n}_1=\mathbf{s}\times\mathbf{e}_x$; $d_1=-\mathbf{n}_1^T\mathbf{x}_A$;
$\mathbf{n}_2=\mathbf{s}\times\mathbf{e}_y$; $d_2=-\mathbf{n}_2^T\mathbf{x}_A$; { see fig.4.c },
end;

end;

Algorithm 4

It can be seen that for case ii = 1 we get

$$\mathbf{n}_1 = [\ -s_z\ ,\ 0\ ,\ s_x\ ]^T\ , \qquad \mathbf{n}_2 = [\ s_y\ ,\ -s_x\ ,\ 0]^T$$

and then

$$F_1(\mathbf{x}) = \mathbf{n}_1^T\mathbf{x} + d_1 = a_1x + c_1z + d_1 \quad , \qquad c_1 \neq 0$$
$$F_2(\mathbf{x}) = \mathbf{n}_2^T\mathbf{x} + d_2 = a_2x + b_2y + d_2 \quad , \qquad b_2 \neq 0$$

For ii = 2

$$\mathbf{n}_1 = [\ s_y\ ,\ -s_x\ ,\ 0\ ]^T\ , \qquad \mathbf{n}_2 = [\ 0\ ,\ s_z\ ,\ -s_y\ ]^T$$

and then

$$F_1(\mathbf{x}) = \mathbf{n}_1^T\mathbf{x} + d_1 = a_1x + b_1y + d_1 \quad , \qquad a_1 \neq 0$$
$$F_2(\mathbf{x}) = \mathbf{n}_2^T\mathbf{x} + d_2 = b_2y + c_2z + d_2 \quad , \qquad c_2 \neq 0$$

similarly for ii = 3

$$\mathbf{n}_1 = [\ 0\ ,\ s_z\ ,\ -s_y\ ]^T, \qquad \mathbf{n}_2 = [\ -s_z\ ,\ 0\ ,\ s_x\ ]^T$$

and then

$$F_1(\mathbf{x}) = \mathbf{n}_1^T\mathbf{x} + d_1 = b_1y + c_1z + d_1 \quad , \qquad b_1 \neq 0$$
$$F_2(\mathbf{x}) = \mathbf{n}_2^T\mathbf{x} + d_2 = a_2x + c_2z + d_2 \quad , \qquad a_2 \neq 0$$

Further simplification of $F_i(\mathbf{x})$ functions is possible because for case ii = 1

$$c_1 \neq 0 \qquad \text{and} \qquad b_2 \neq 0$$

Therefore we can introduce modified separation functions $F_i^{'}(\mathbf{x})$ i = 1, 2 as

$$F_1^{'}(\mathbf{x}) = F_1(\mathbf{x}) / c_1 = a_1^{'}x + z + d_1^{'} \quad , \qquad c_1 \neq 0$$
$$F_2^{'}(\mathbf{x}) = F_2(\mathbf{x}) / b_2 = a_2^{'}x + y + d_2^{'} \quad , \qquad b_2 \neq 0$$

and similarly for other cases.

This modification speeds up the proposed algorithm as two additions and four multiplications are saved for each facet. Precomputations of $F_i^{'}(\mathbf{x})$ functions are made for each clipped line only. That is convenient as the number of tested facets of the given polyhedron is expected to be high (for a sphere approximation might reach 10^4).

3. Theoretical analysis

For considering efficiency of any algorithm we have to take into account at least floating point operations and their timing, see tab.1. Before making any comparisons it is necessary to point out that time needed for each operation (:= , < , ± , * , /) does differ from computer to computer.

operation	:=	<	±	*	/
float	33	50	16	20	114
int	5	9	3	26	44

Time is in 1/10 sec for 10^5 operations for PC 486/33 MHz
Table 1

Let us denote N number of facets of the given convex polyhedron. Then it is possible to express the time complexity T_{CB} of the CB algorithm, see alg.1, as

$$T_{CB} = (6 , 3 , 4 , 9 , 1) * N$$

and computational time can be estimated using tab.1 as

$$T_{CB} = 706 * N$$

The time complexity T_0 of the algorithm that directly computes the intersection point of the given line *p* with a triangle using parametric equations can be estimated as

$$T_0 = (5.5 , 2.5 , 18 , 15 , 1.5) * N$$

and using tab.1 we obtain computational time

$$T_0 = 1065.5 * N$$

It means that this algorithm will be slower than CB algorithm.

The time complexity T of the proposed algorithm can be estimated

- for the worst case as $T = (2 , 3.5 , 12 , 12 , 0) * N$

- for an average case as $T = (2 , 2 , 6 , 6 , 0) * N$

Using tab.1 we can evaluate the expression for T as

$T = 673 * N$ for the worst case,
$T = 382 * N$ for an average case

Let us introduce coefficients of efficiency as

$$\nu_1 = \frac{T_{CB}}{T} \qquad \nu_2 = \frac{T_0}{T}$$

where: T_{CB} , T_0, T are execution times needed by the CB algorithm, algorithm that uses parametric equations and the proposed algorithm (CLIP_3D_MOD).

Now it is possible to estimate the asymptotic behaviour of the proposed algorithm based on theoretical complexity estimations:

- for the worst case as $\nu_1 = \frac{T_{CB}}{T} = \frac{706 * N}{673 * N} = 1.04$

- for an average case as $\nu_1 = \frac{T_{CB}}{T} = \frac{706 * N}{382 * N} = 1.85$

The obtained theoretical estimations and comparisons show that the proposed algorithm should be faster to the CB algorithm significantly.

4. Experimental results

The proposed algorithm has been tested against Cyrus-Beck's algorithm and algorithm that uses parametric equations. Data sets of two points that define a line have been randomly and uniformly generated inside a sphere in order to eliminate an influence of rotation. More than 10^4 randomly generated lines were used. Convex polyhedra were generated as N- sided convex polyhedra that consist of triangles and were inscribed into a smaller sphere.

Results obtained from experiments are shown in tab.2 and tab.3 for two fundamental cases when no line intersects the given polyhedron (0%) and when all lines intersect (100%) the given polyhedron. In general case the efficiency depends approximately linearly on the percentage of lines that intersect the given polyhedron. All tests were made on PC 486/33 MHz.

N	8	48	224	960
0 %	1.01	0.92	1.18	1.22
100 %	0.5	0.79	1.08	1.61

Efficiency coefficients ν_1 if condition j < 2 **was not used**
Table 2

N	8	48	224	960
0 %	1.41	2.08	2.13	2.23
100 %	1.07	1.58	1.90	2.15

Efficiency coefficients ν_2 if condition j < 2 **was not used**
Table 3

The efficiency of the proposed algorithm against CB algorithm for $N \geq 224$ is

$\nu_1 \in <1.08\ ,\ 1.66>$ (condition j < 2 was not used)
$\nu_1 \in <1.64\ ,\ 2.49>$ (condition j < 2 was used)

and the efficiency of the proposed algorithm against direct intersection computation

$\nu_2 \in <1.9\ ,\ 2.23>$ (condition j < 2 was not used)
$\nu_2 \in <1.5\ ,\ 2.9>$ (condition j < 2 was used)

It means that the proposed algorithm for $N \geq 224$ was always faster than the CB algorithm.

In the course of the algorithm experimental evaluation was found that the proposed algorithm is sensitive to implementation of the sign(x) function. We actually need just determine if $x \geq 0$. In that case it is possible just to test a sign bit of the float point representation of the value x.

The proposed algorithm can be generalised for a nontriangular facets by replacing procedure SOLVE by a more general procedure or by using a similar approach as the CB algorithm does, e.g. use the CB algorithm just for all facets that were intersected by both planes ρ_1 and ρ_2. In case that normal vectors of all facets can be precomputed we can obtain an additional speed up of the proposed algorithm if convex polyhedron is considered and all facets are oriented.

Tab.4 shows the final experimental results if sign(x) function and all sign comparisons were implemented carefully and the SOLVE procedure was implemented as a modification of the CB algorithm for the case when normal vectors for all facets were precomputed, see alg.5.

N	10	20	100	200	500
0 %	1.28	2.08	2.69	2.89	3.17
100 %	0.3	0.57	1.57	2.06	2.67

Efficiency coefficients ν_1 if condition j < 2 **was not used**
Table 4

```
procedure CLIP_3D_EFF ( x_A, x_B );
begin  t_min := 0.0 ;      t_max := 1.0 ;
       s := x_A − x_B;            i := 1;          j := 0;
       { for a line initialization: t_min := −∞ ; t_min := ∞ ; }
       { ρ1: a1x + c1z + d1 = 0                  ρ2: b2y + c2z + d2 = 0 }
       for k := 1 to N_V do { N_V number of vertices }
              Q := sign(F1(x_k)); { Q is a vector of int or char types }
       while (i <= N) and (j < 2) do
       begin  { ^i x_k means a k-th vertex of the i-th triangle }
              { INDEX(i,k) gives the index of k-th vertex }
              { of the i-th triangle, i.e. ^i x_k = x_INDEX(i,k) }
              if Q_INDEX(i,0) = Q_INDEX(i,1) then
                     if Q_INDEX(i,0) = Q_INDEX(i,2) then goto 1;
                            { do nothing ρ1 does not intersect the i-th triangle }
              if sign(F2(x_INDEX(i,0))) = sign(F2(x_INDEX(i,1))) then
                     if sign(F2(x_INDEX(i,0))) = sign(F2(x_INDEX(i,2))) then goto 1;
                            { do nothing ρ2 does not intersect the i-th triangle }
              { both planes ρ1, ρ2 intersect the i-th triangle }
              { one step of the CB algorithm }
              { ------------------------------------------------ }
              { n_i is a normal vector of the i-th facet }
              { and n_i must point out of the convex polyhedron }
              { v_j is the j-th vertex of the polyhedron }
              ξ := s^T n_i ;      s_i := x_i − x_A ;
              if ξ ≠ 0.0 then
              begin  t := s_i^T n_i / ξ;
                     if ξ > 0.0 then t_max := min(t, t_max) else t_min := max(t, t_min)
              end
              else { line is parallel to the i-th facet } Special case solution;
1:            i := i +1;
       end { while };
       if t_min ≤ t_max then SHOW_LINE( x(t_min), x(t_max) );
end { of CLIP_3D_EFF };
```

Algorithm 5

5. Modification for NonConvex Polyhedra

It can be seen that the proposed algorithm can be used for clipping lines against nonconvex polyhedra, too. For this case only the condition j < 2 must be removed from the

while statement in the suggested algorithm, see alg.3., and the lines marked by (*) must be replaced by a sequence, see alg.6. Of course, it is necessary to sort all values t. Intervals $< t_{2j-1}, t_{2j} >$ define parts of the given line which lies inside of the given nonconvex polyhedron.

```
SORT_ALL_VALUES ( { ti } );
for k := 1 to j/2 do
        if < t2k-1, t2k > ∩ < 0,1 > ≠ ∅ then  SHOW_LINE( x(t2k-1) , x(t2k) );
```

Nonconvex case modification for alg.3.
Algorithm 6

If line clipping is considered instead of line segment then the part $\cap < 0,1 > \neq \emptyset$ the **if** statement has to be left out.

The complexity of the proposed modified algorithm in this case will be

$$O(N) + O(k \lg k)$$

where: k is a number of intersections of the given line and the given polyhedron (for situation on Fig.1.b is k = 4),
N is a number of triangles (facets).

In experimental algorithm verification nonconvex polyhedra were generated by a special 3D graphics editor which enabled to prepare quite complex objects. In the case of **nonconvex** polyhedron direct solution of parametric equations were only used and the efficiency coefficient ν_2 **is always higher than one** for all N. All experiments showed that

$$\nu_2 \in < 1.07 \ , \ 2.15 >$$

6. Conclusion

The new efficient algorithm of $O(N)$ complexity for clipping lines against convex polyhedron was developed with modification for nonconvex polyhedron case, too. The proposed algorithm does not strictly require triangular facets and can be easily modified for nontriangular facets using a different SOLVE procedure for general polygonal facets. The given facets should be oriented. All tests were implemented in C on PC 486/33 MHz.

7. Acknowledgments

The author would like to express his thanks to students of Computer Graphics courses at the University of West Bohemia in Plzen and Charles's University in Prague for their suggestions and critical comments that stimulated this project, especially to P.Sebránek and J.Jirák for verifications of all tests and implementation of the proposed algorithm, to P.Bláha for making the 3D graphics editor for preparing polyhedra.

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