Tiling

Ivana Kolingerová

1. Golden ratio
2. Tiling
3. Celtic ornaments

References:

Andrew Glassner's Notebook:
Aperiodic Tiling, Penrose Tiling, Celtic Knotwork I-III,
IEEE Computer Graphics and Applications, 1998-2000

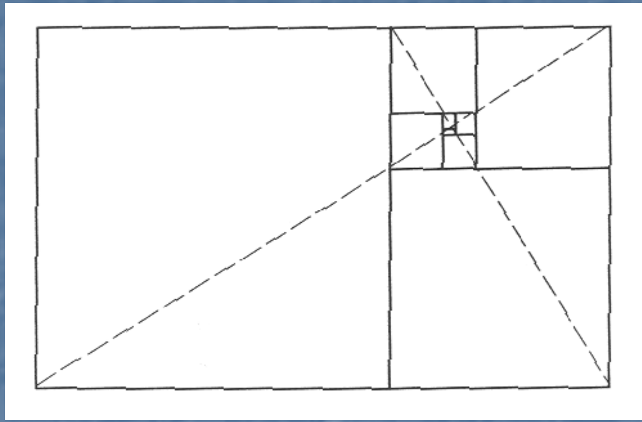
https://en.wikipedia.org/wiki/Golden_ratio

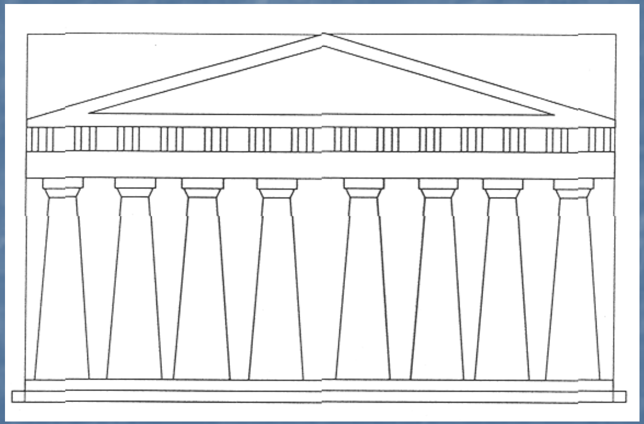
1. Golden ratio

Golden rectangle: sides Φ, 1, not too thick, not too thin

We can find in the Greek Parthenon, Mona Lisa, Dalí, Escher ...

When we cut out the square, we have again a golden rectangle





$$\phi = 1 + \frac{1}{\phi} = \frac{1+\sqrt{5}}{2} \cong 1.618033989..$$

$$\frac{1}{\phi} = 0.618033989$$

$$\phi = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+...}}}}$$

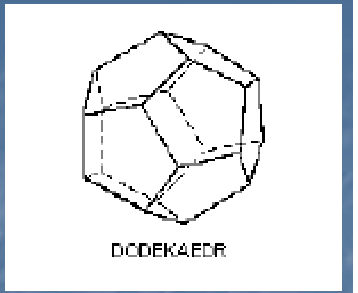
$$\phi = 1 + \cfrac{1}{1+\cfrac{1}{1+...}}$$

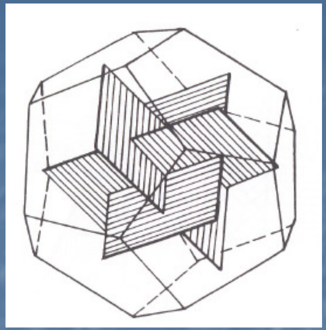
Sometimes Φ and 1/Φ reversed

1/ Φ – the rate of edge lengths of a golden rectangle, see fig.

In fact an infinite picture regression ...

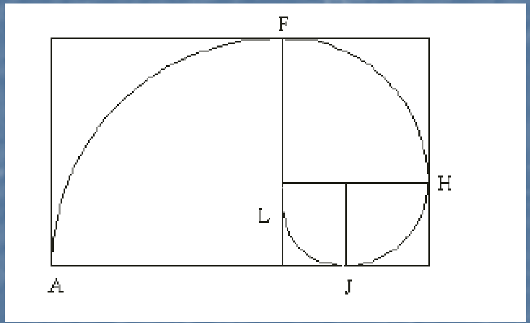
Another Φ computation: the rate of two following Fibonacci numbers





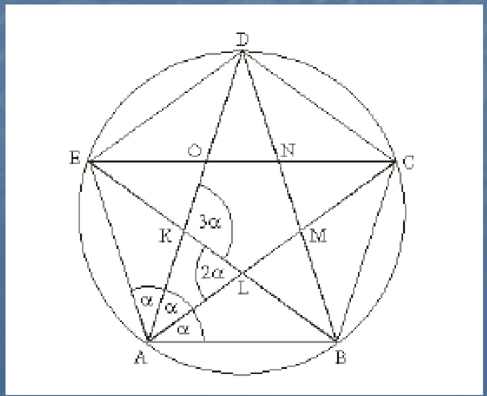
Regular dodecahedron

- 3 mutually rectangular „golden“ rectangles can be inscribed



Logarithmic spiral

- Follows golden ratios

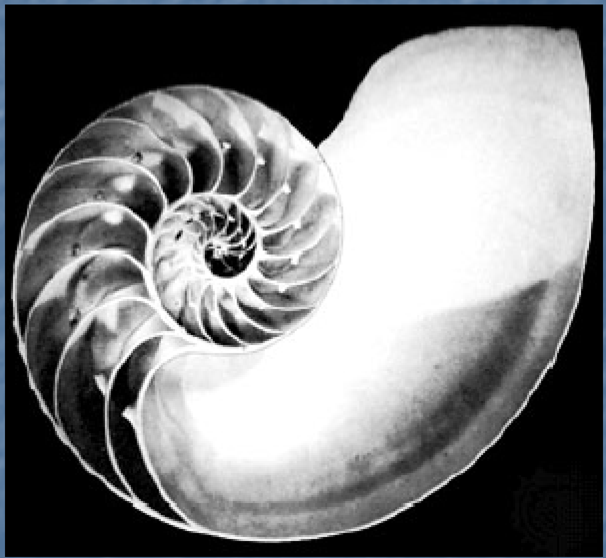


Pentagram is a rich source of golden ratios

Diagonals cut in golden ratios
- the rate of a diagonal and an edge is golden
- diagonals form 5-star,
  inside again a regular pentagram

Golden ratio in widelife

- Logarithmic spiral – the growth of various inorganic parts (beaks, horns, teeth, tusks, conches ...)



Golden ratio for composition of photos or pictures

Central composition

- Static, calm, sometimes even boring, the central object might have too much space around

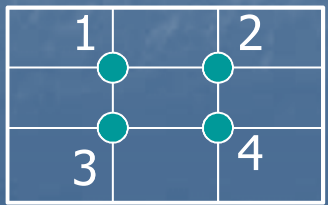
Altán ve středové kompozici

Golden ratio

- Uses the direction of human view of an image
- OK to divide in about 1/3

Altán ve zlatém řezu

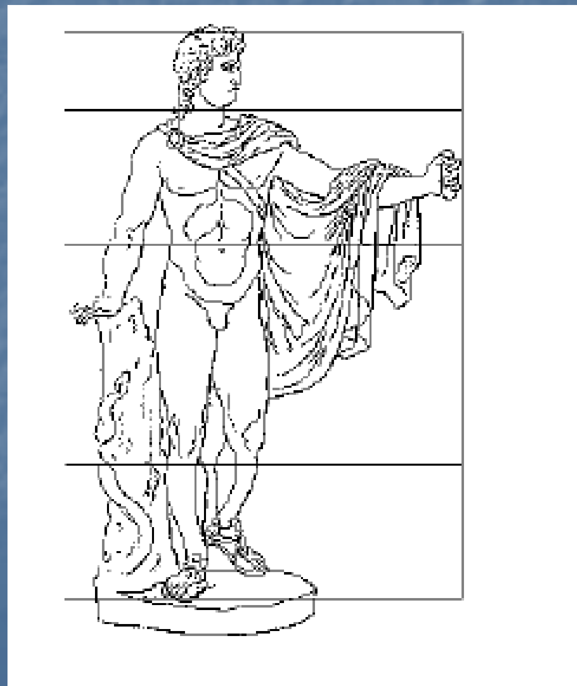
The direction of view



Gold ratio in art

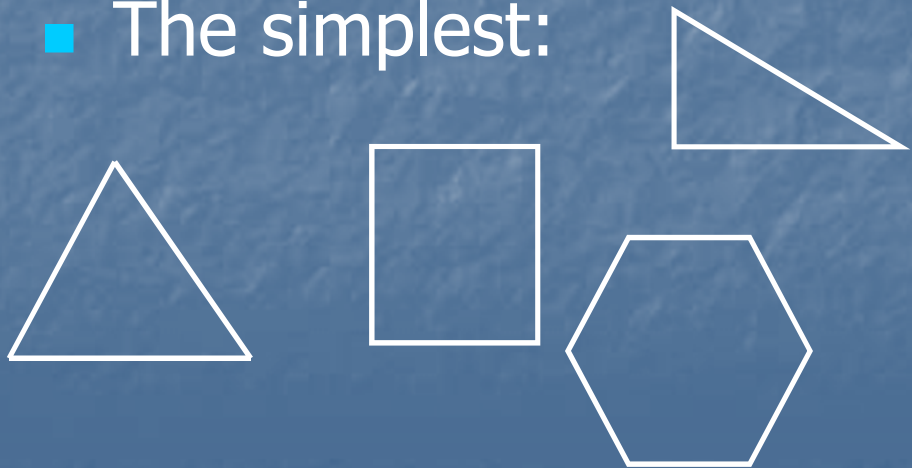
- Most often used in reneissance
- Picture format – „golden“ rectangle in both orientations
- Placement into the golden ratio

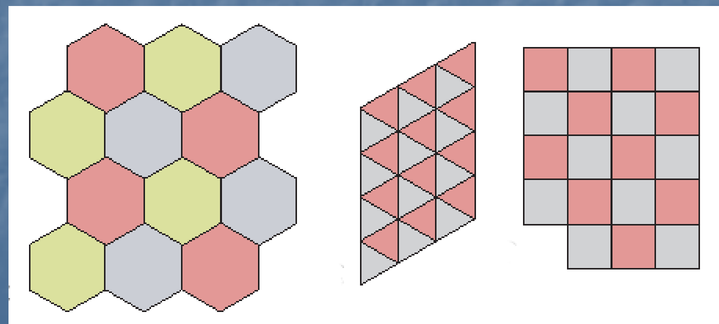
- Construction of human body model by golden ratio (rate of lengths above and below the waist, these parts can be again subdivided in golden ratio)



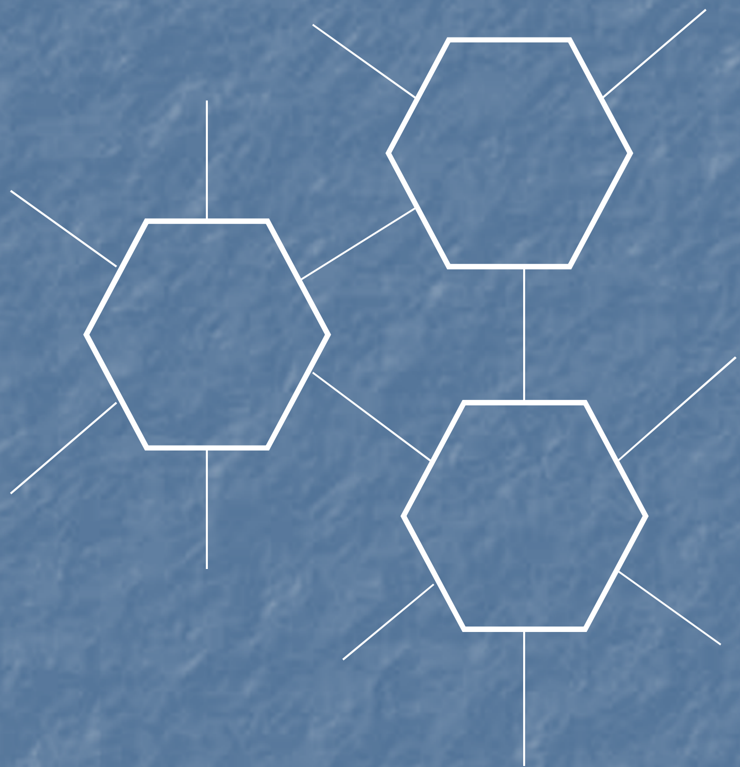
2. Tiling

- Regular patterns pleasant for human vision
- But: too regular - boring
- Artists utilize a tension between regularity and surprise
- Patterns: a small set of figures repeated in the whole plane – a tiling, a tessellation
- The simplest:



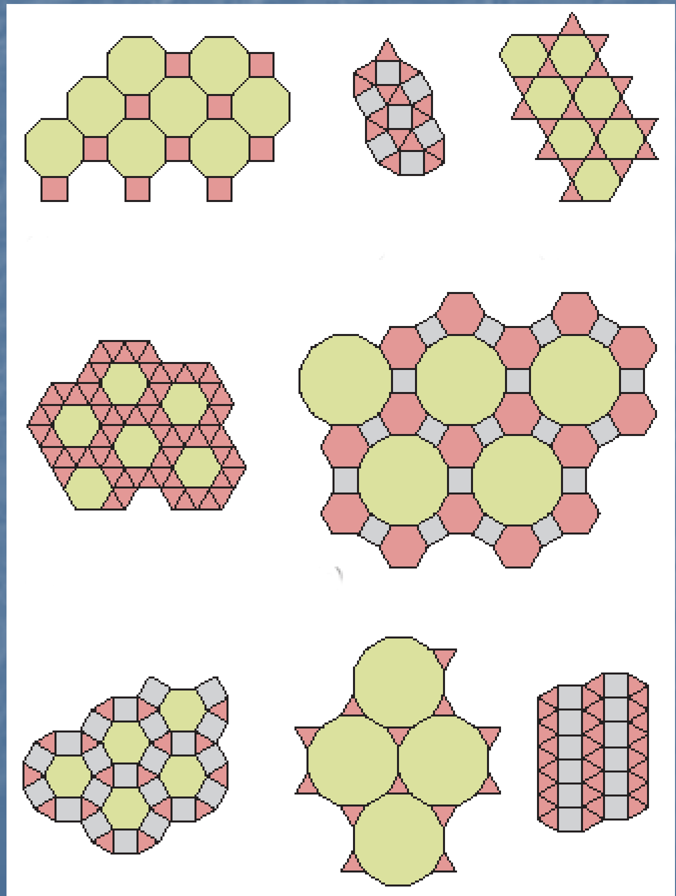


- Variations:



- Usage: networks, VLSI design for memories (6gons – processors)

- Semiregular patterns – consisting of more than 1 type of polygon
- In each vertex
  the same polygon types
  in the same order

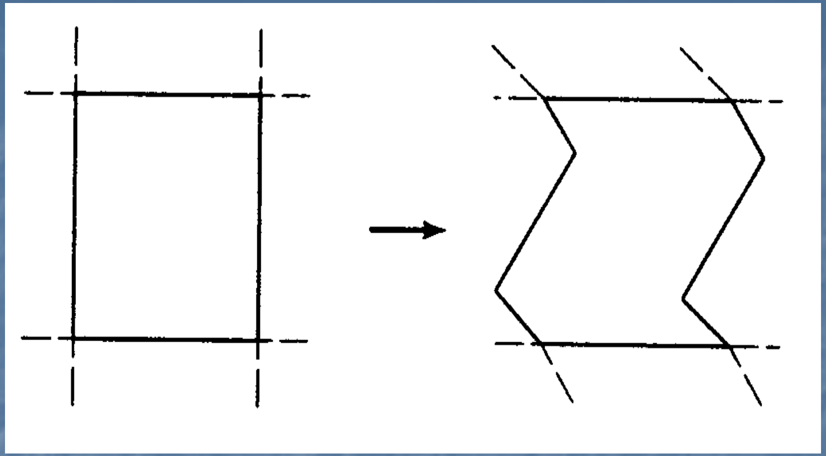


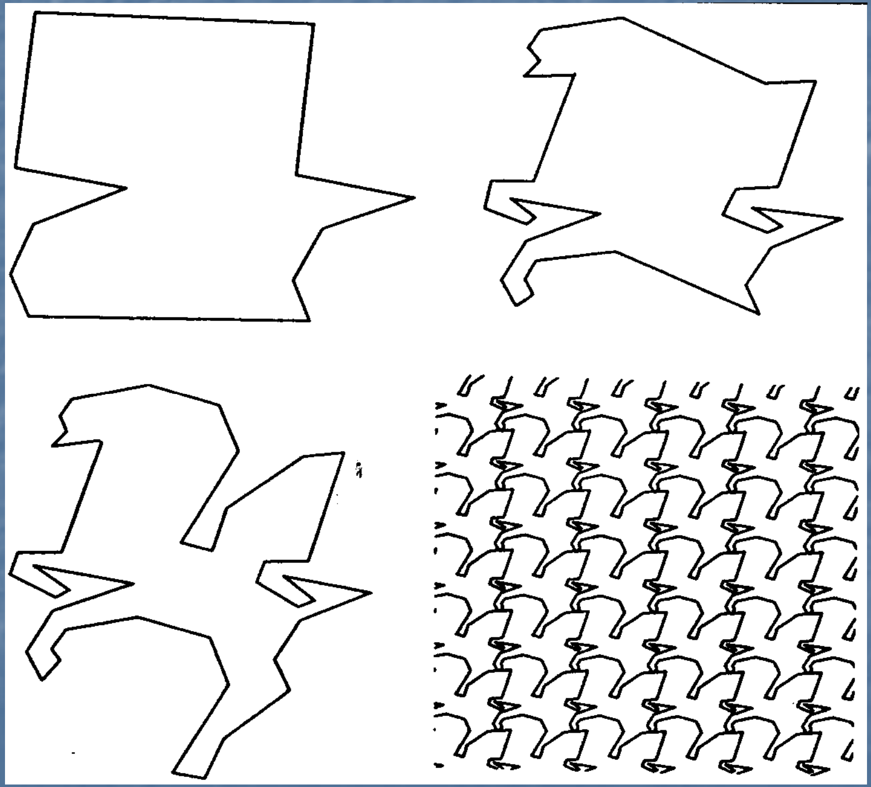
A simple tessellation:

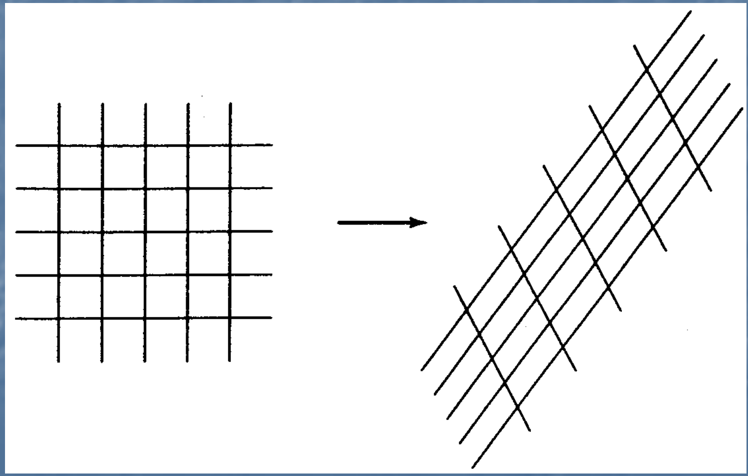
```
for i :=0 to NumRows-1 do
    begin
     if Odd(i) then Offset := shift
     else Offset := 0;
     for j := 0 to NumCols-1 do
        begin
          Triangle (j*ColWidth + Offset,
                    i*RowWidth,1)
        end
    end
```

More general:

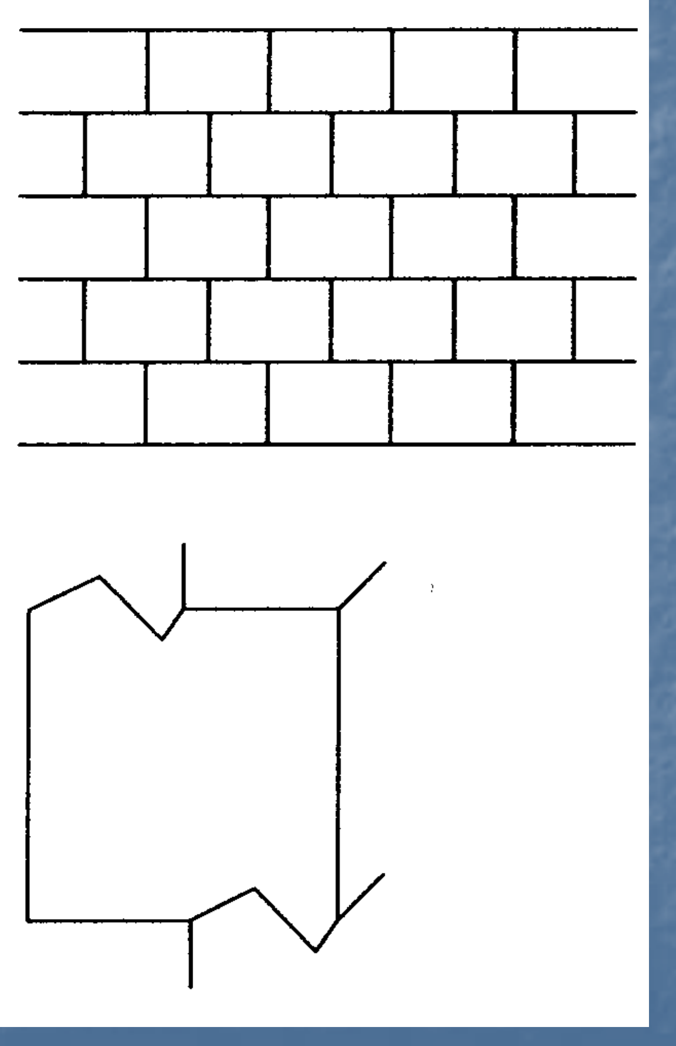
- Deformation of a square



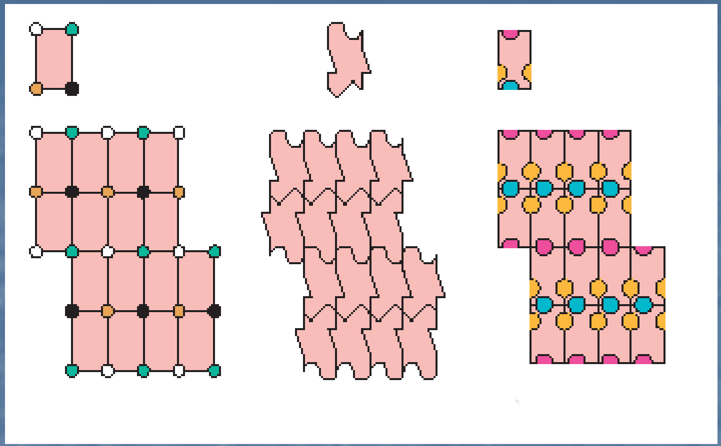


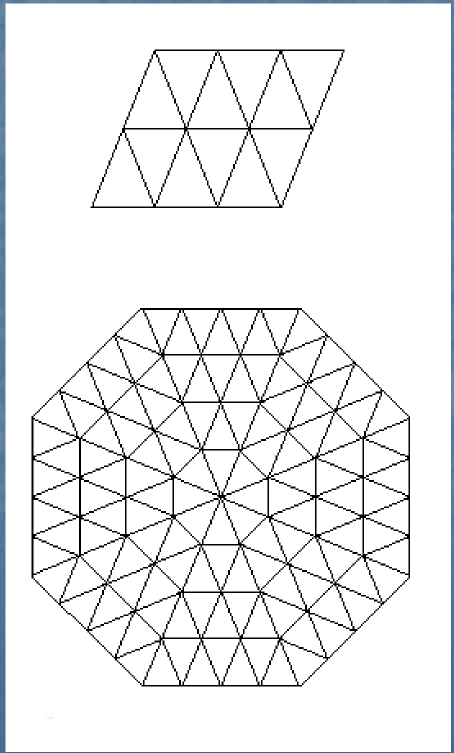


- Brick deformation
- Deformations
  on the opposite sides
  must correspond



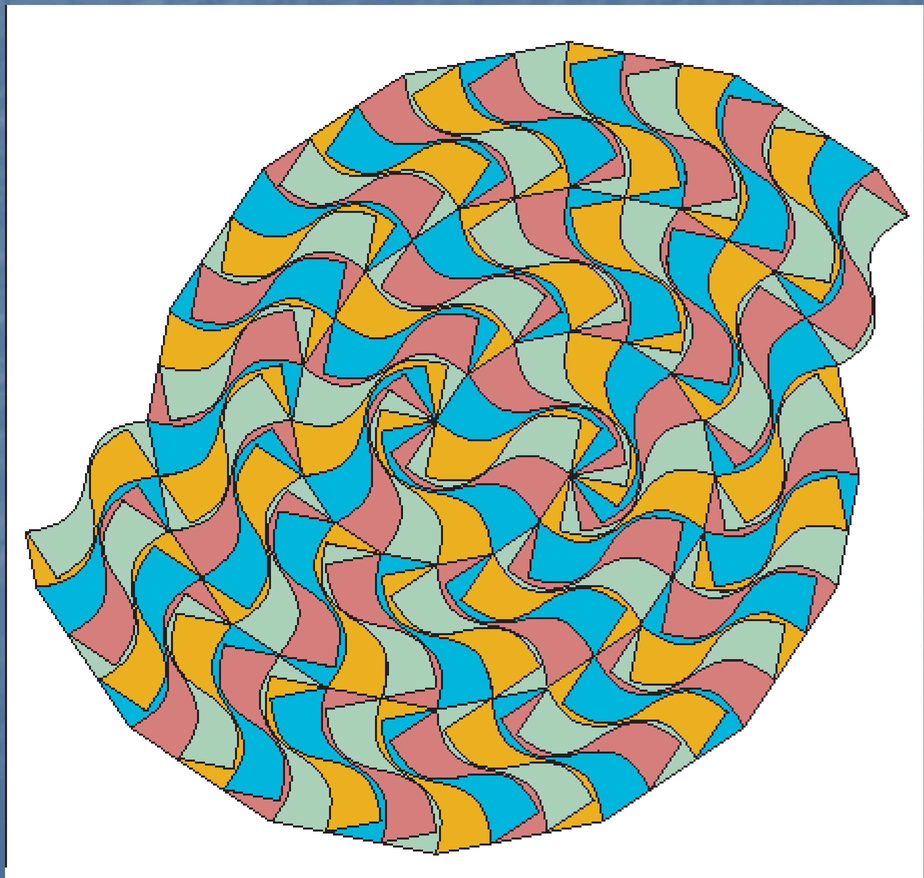
- Tiling
  - Periodic -
    incidence of
    corresponding vertices,
    edge incidence,
    connection of face decorations,
    created by a translation
  - Aperiodic



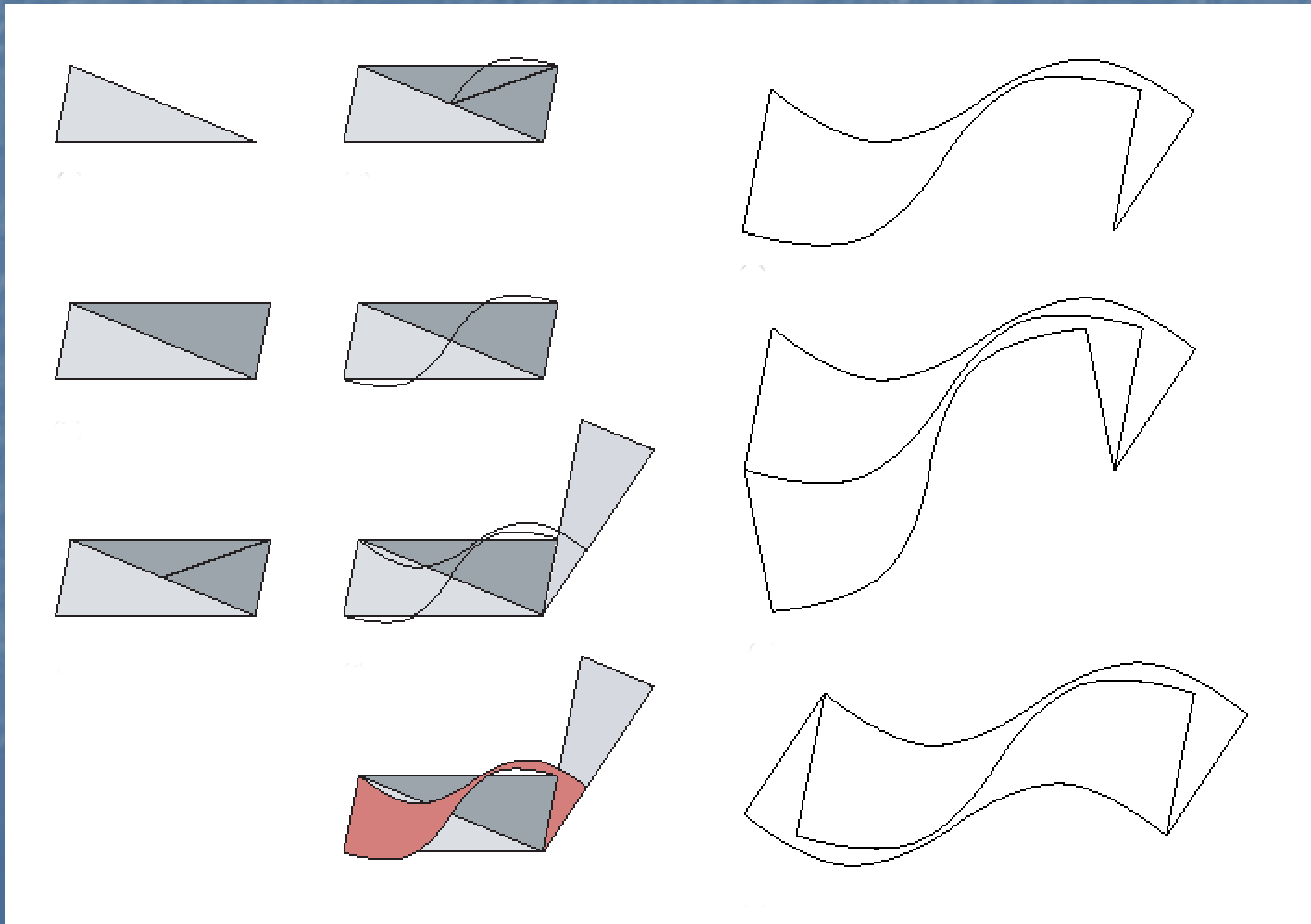


Triangles in periodic
and aperiodic tiling

- An example of an aperiodic tiling (1 type of tile)

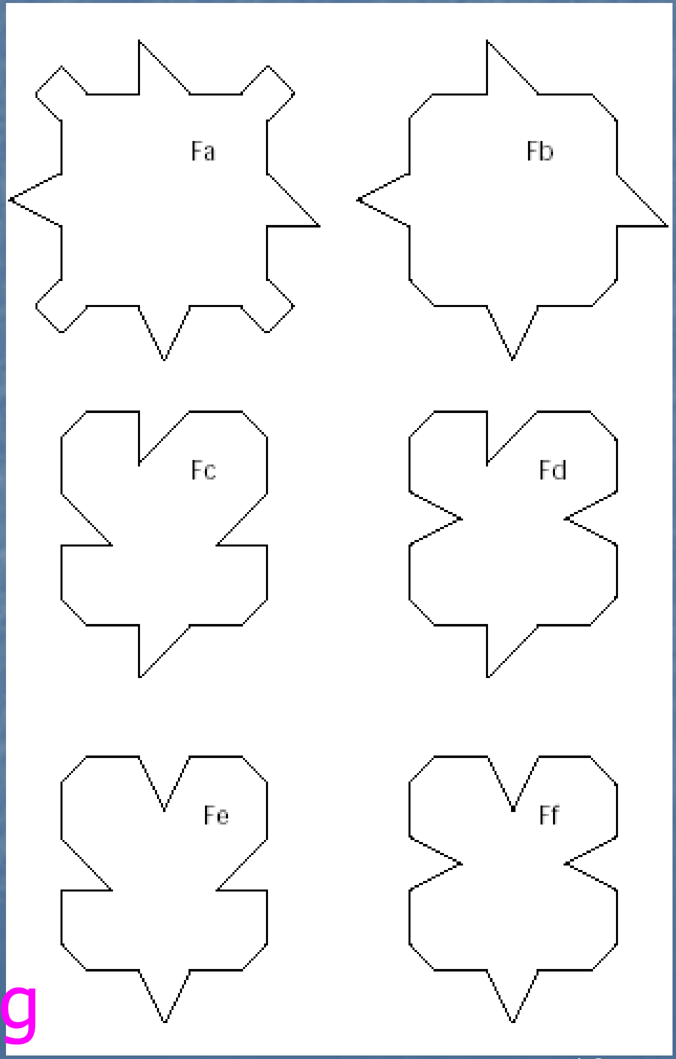


- The tile for the previous example – fits together either after a rotation or by a vertical reflection



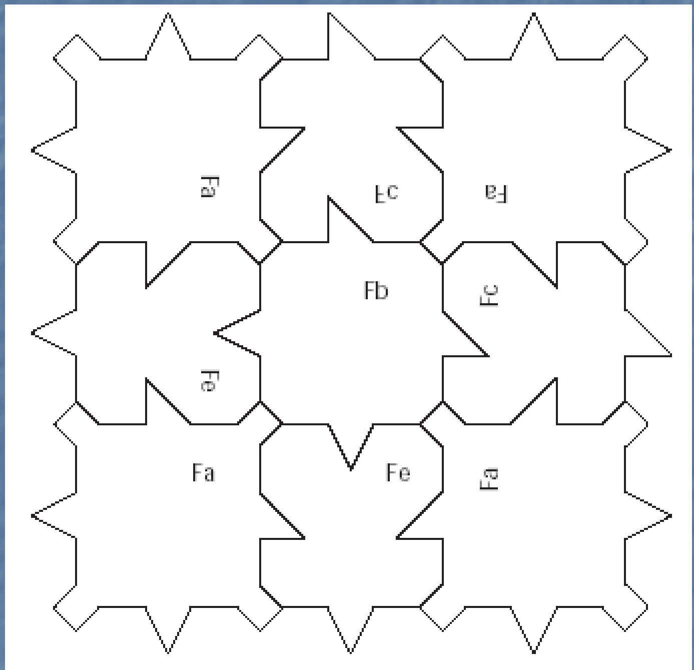
Aperiodic tiling

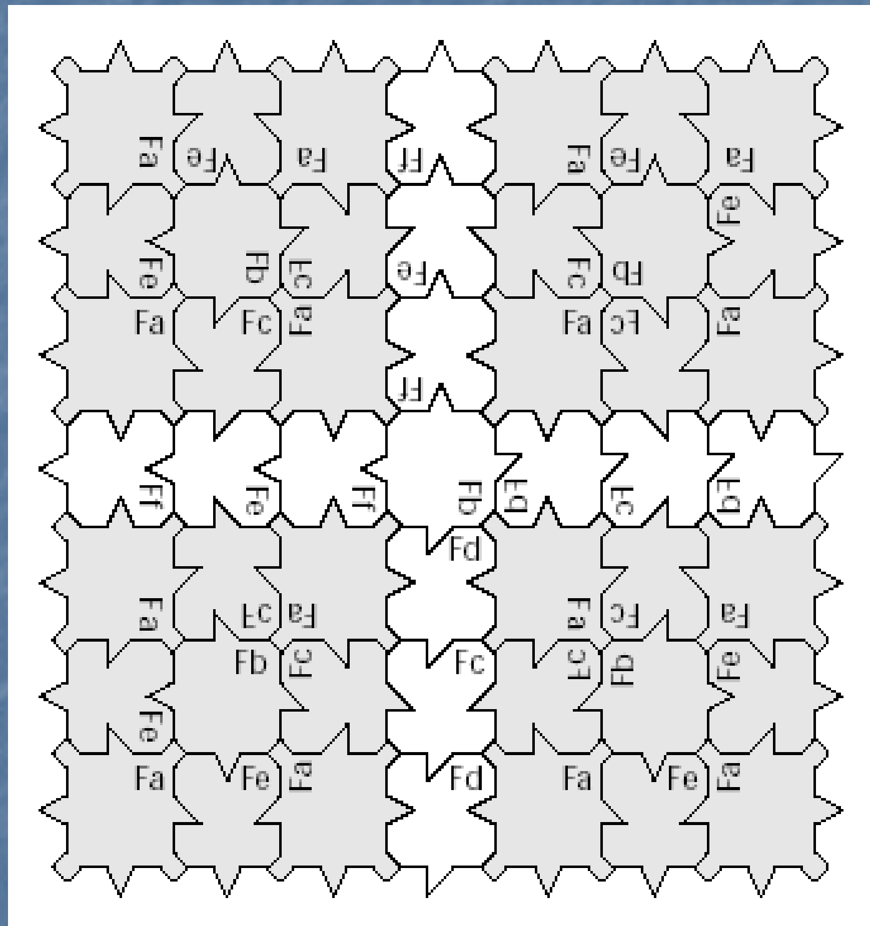
- Symmetry is safe and boring
- More types of tiles, together aperiodic patterns
- Seeked dozens of years, the first one: 26,000 of tile types
- Now: several (according to the type of tiling, at least 2)



Robinson tiling

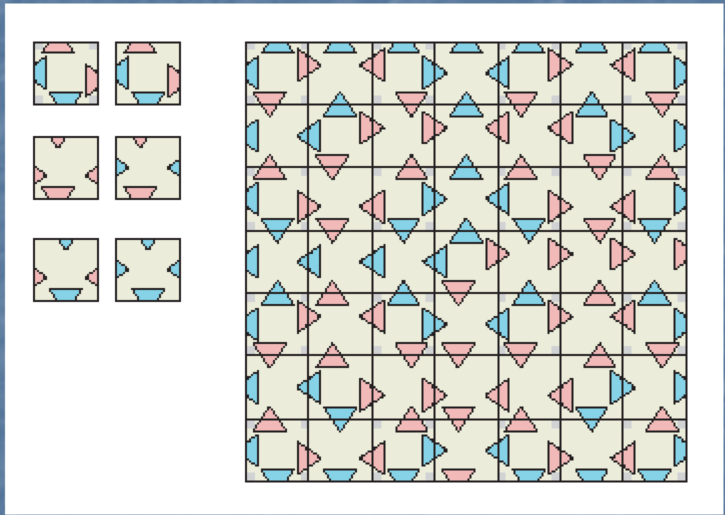
- Use: 3x3 and 7x7



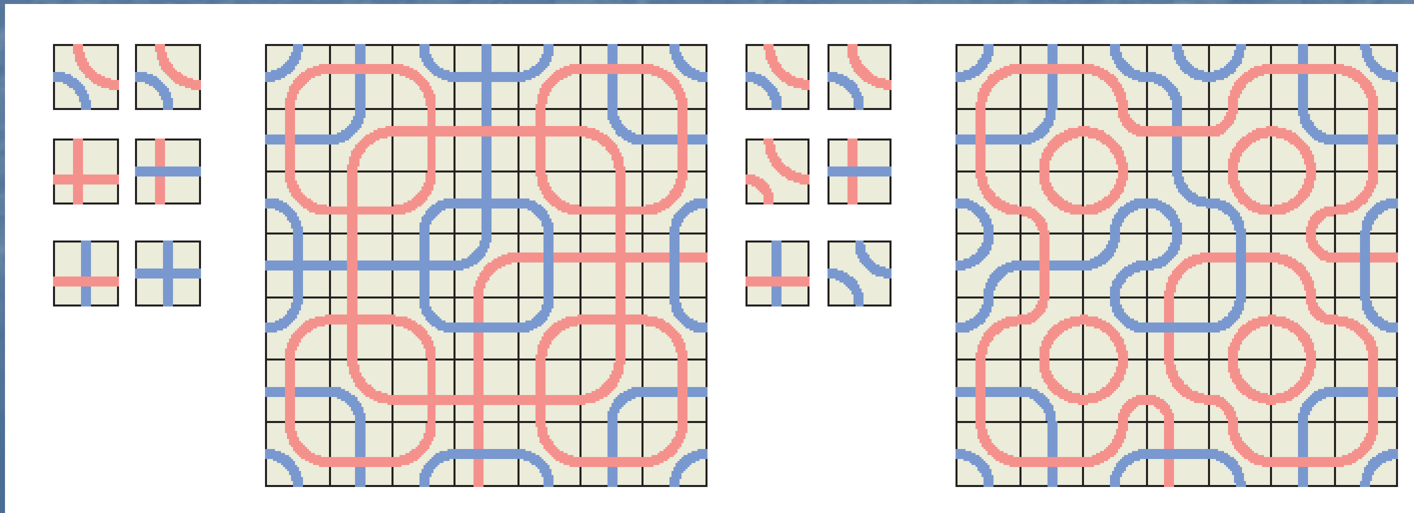


“nearly periodic"

- Or with decorations

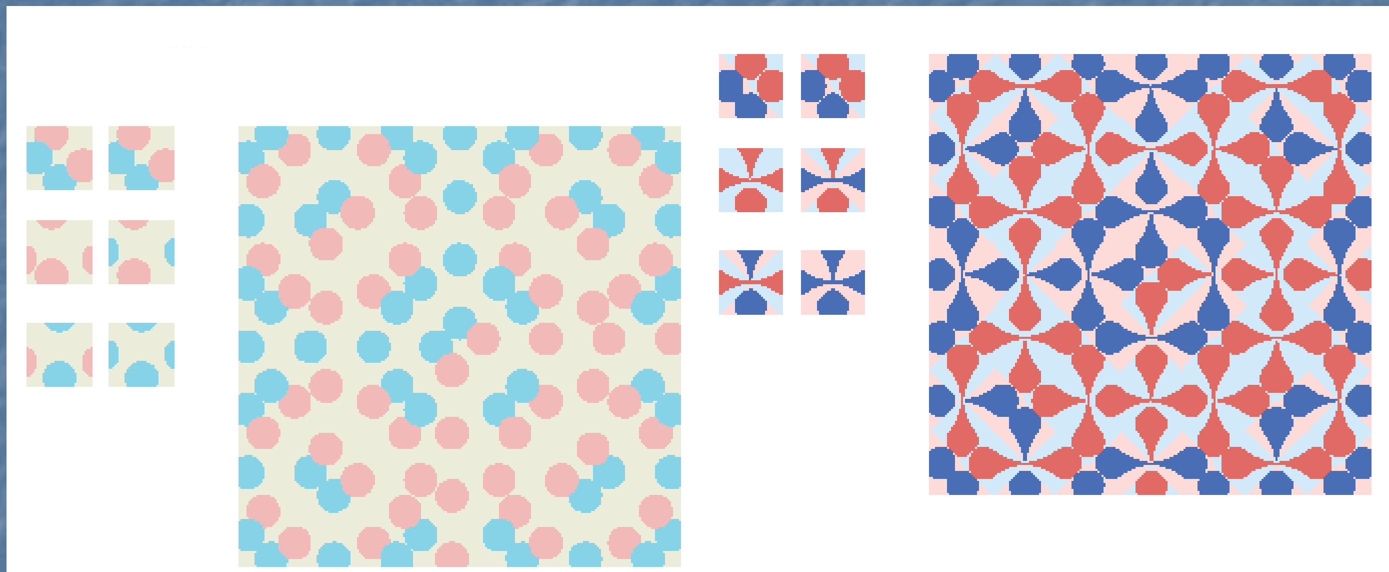


Decorated set 7x7

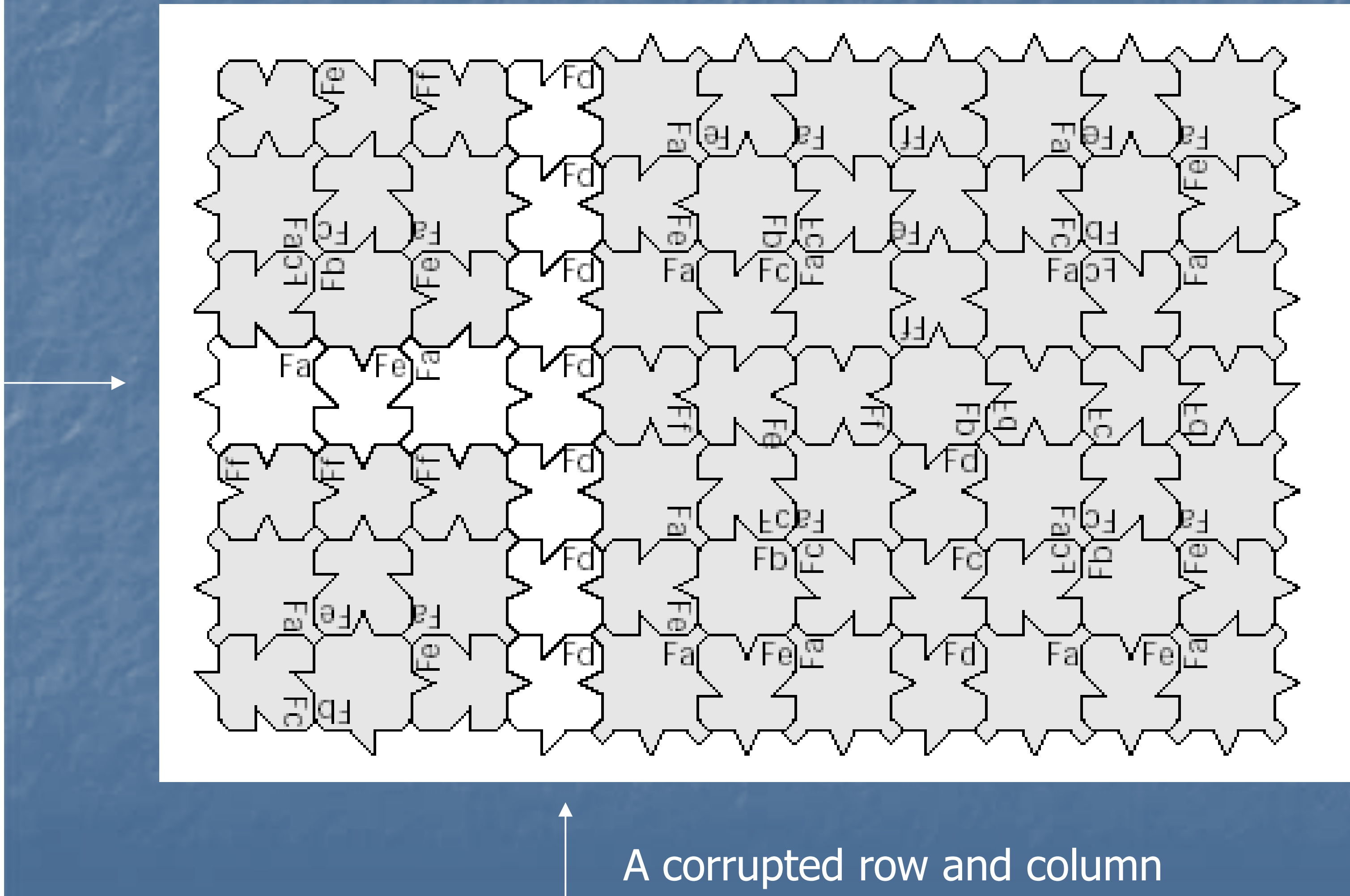


- Other possibilities

Decorated set 7x7

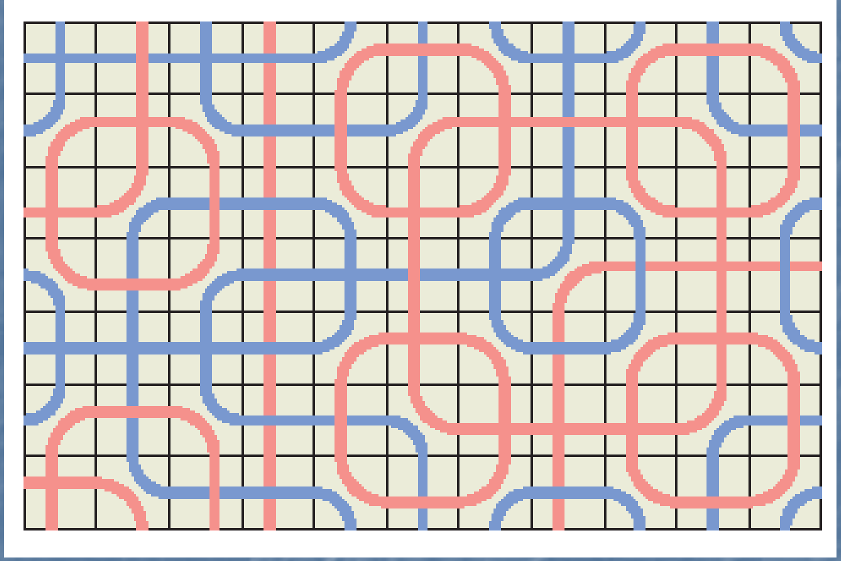


- With intentionally corrupted pattern – e.g., during connection of tiled polygons



A corrupted row and column

- In a decorated version

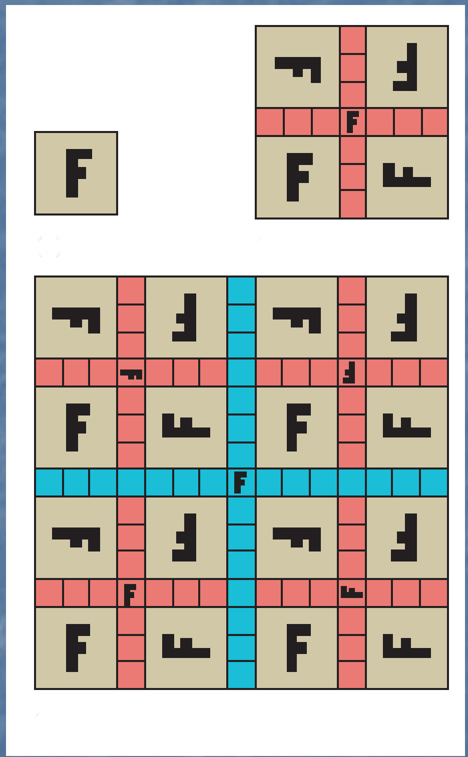


Decorated set with a corruption

- Robinson cannot be set only by translations – aperiodic (look not only at patterns but also at the shape)

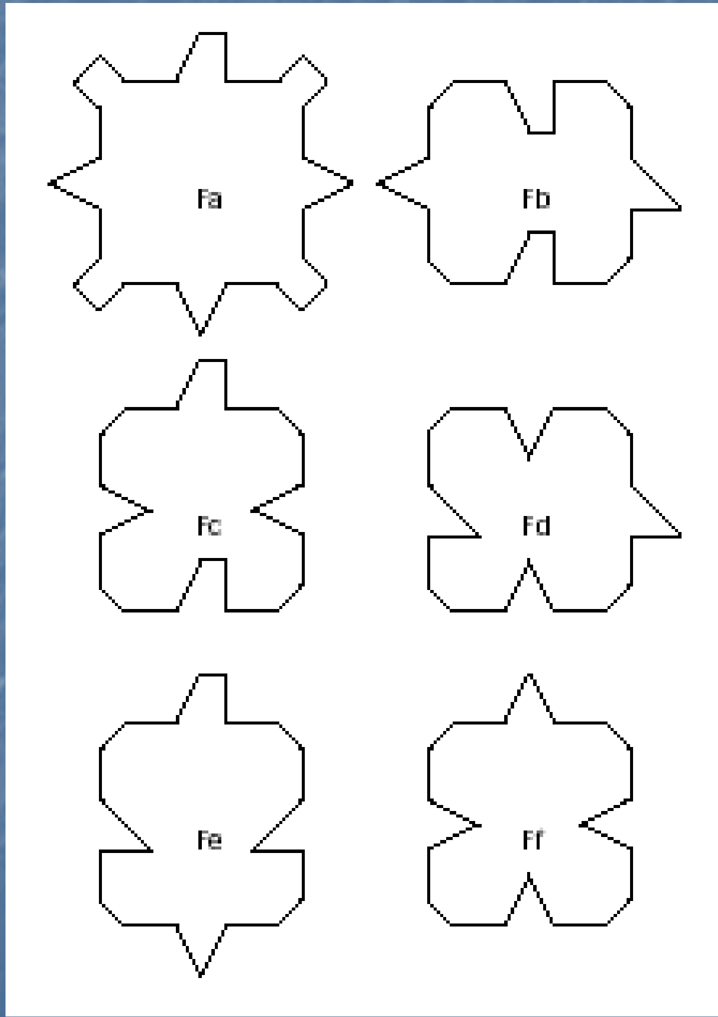
block 3x3

block 7x7 (rotations needed)

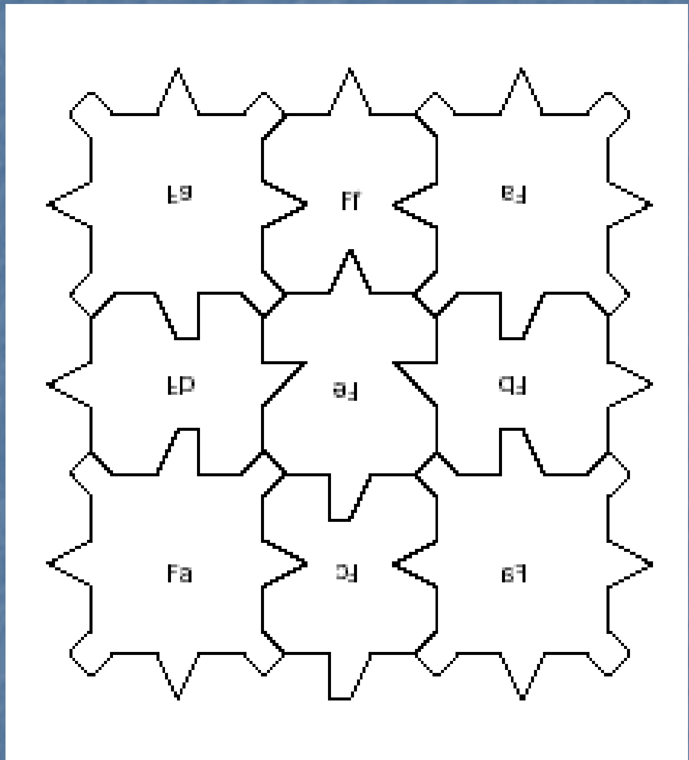


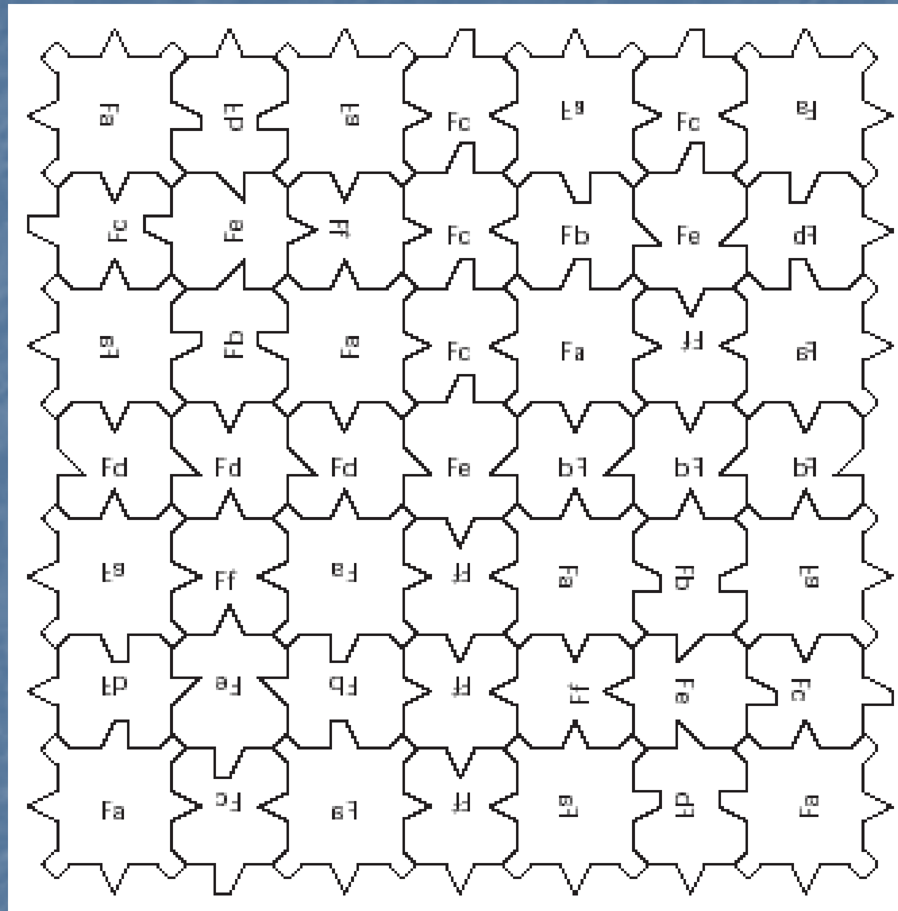
block 15x15 (rotations needed)

- Ammann tiling



- Use: 3x3 and 7x7



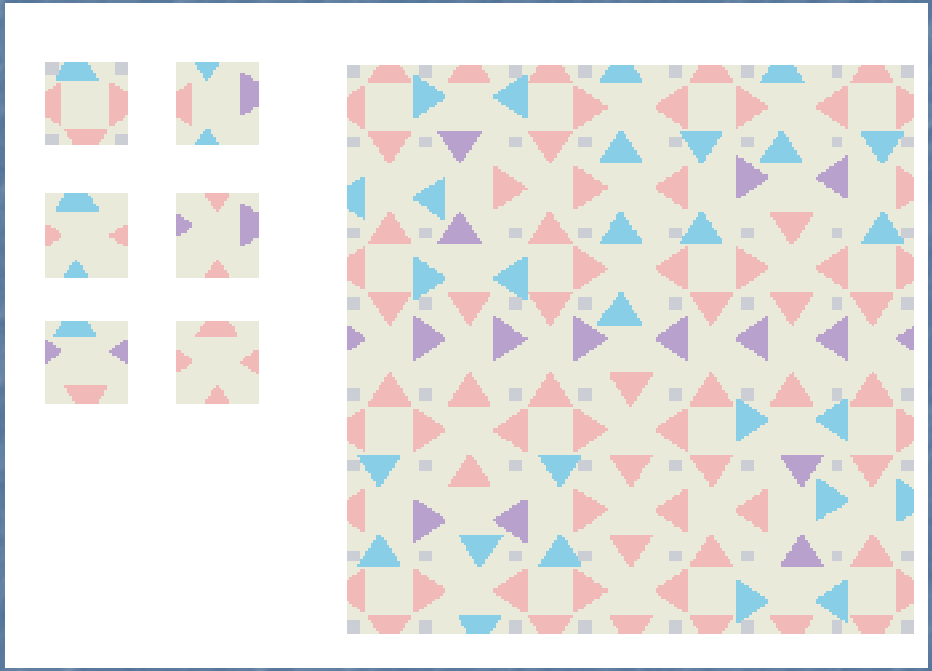


"nearly periodic"

- Similar results to Robinson

Decorated 7x7

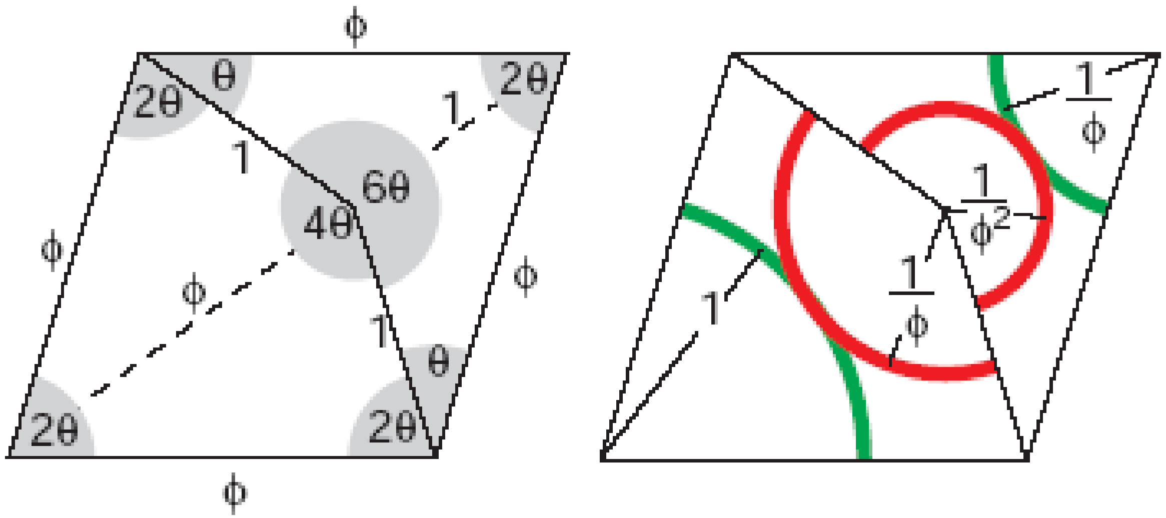
etc.



- Use of Rob. and Ammann besides esthetic images: non-uniform samples for stochastic sampling in, e.g., distributed ray tracing

- Penrose tiles

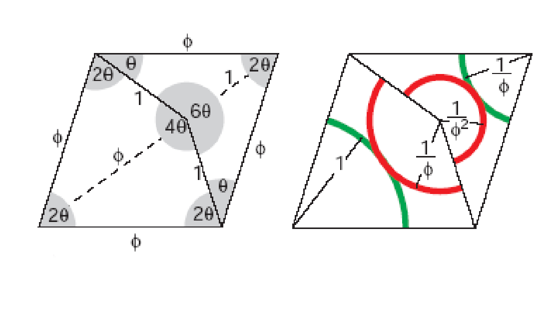
2 types of tiles, "kite" (the bigger) and "dart" (the smaller)

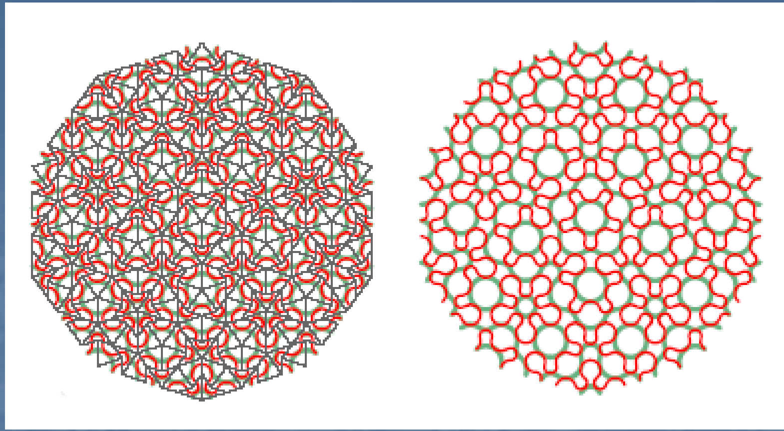


Φ – golden ratio,
θ – angle 36°

Face decoration, arcs must correspond in the tiling

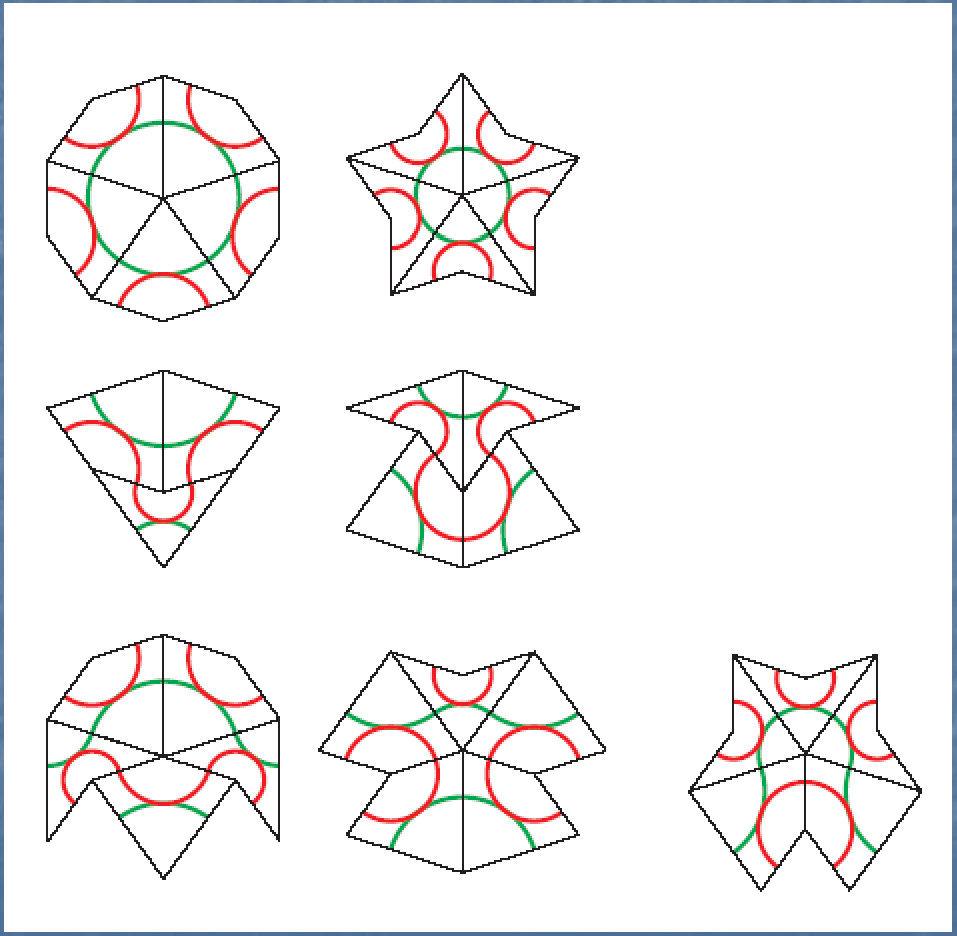
- The lengths of neighbouring sides must be the same and arcs of neighbouring sides must correspond



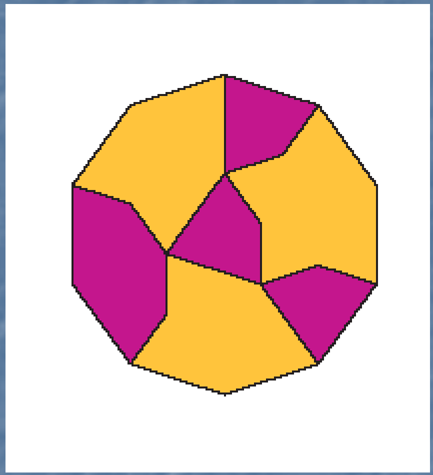


- Symmetrical – if done as real objects, it is enough to decorate one side
- 1.6 times more of darts than kites is needed
- It is easy to deadlock
- To produce the tiling manually is relatively difficult (about 100 pieces in hand – a good work)

- Dictionary of possible sets

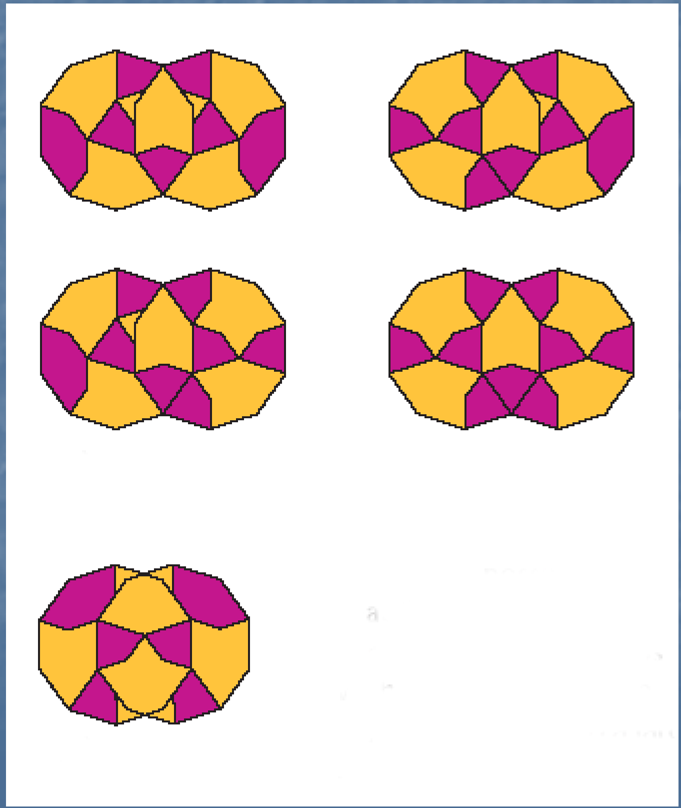


- An alternative: overlap allowed



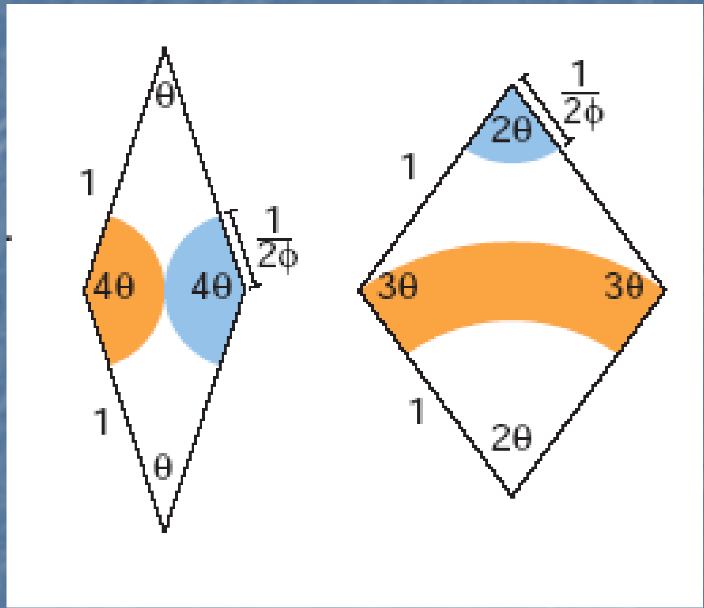
Overlapping of these tiles
produces Penrose tiling

How to do it
(overlapped areas must be
the same colour)



- Similar structures
(so called quasicrystals)
in materials

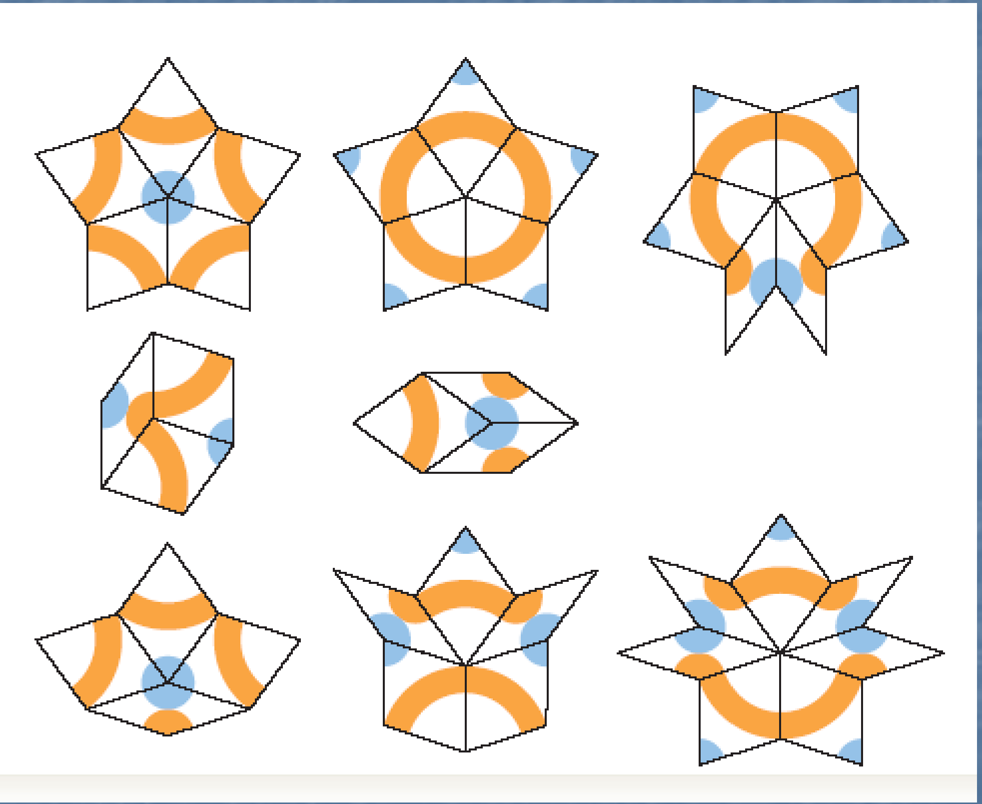
- Penrose kosočtverce



Φ – golden ratio,
θ – angle 36°
All sides of the same length

- Arcs must correspond

- Dictionary of possible sets



Implementation

- For example to divide into triangles, triangles are recursively replaced by new ones, the result is either further subdivided or joined to darts and kites

Further applications:

We want a growing town – not 100% planned in advance

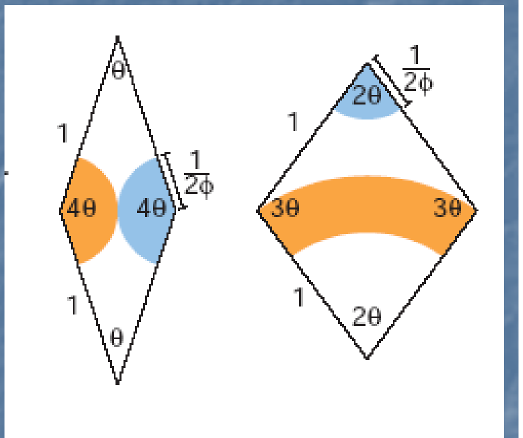
=> Tiles with the decorations of building grounds, used to tile the plane, on them buildings are erected

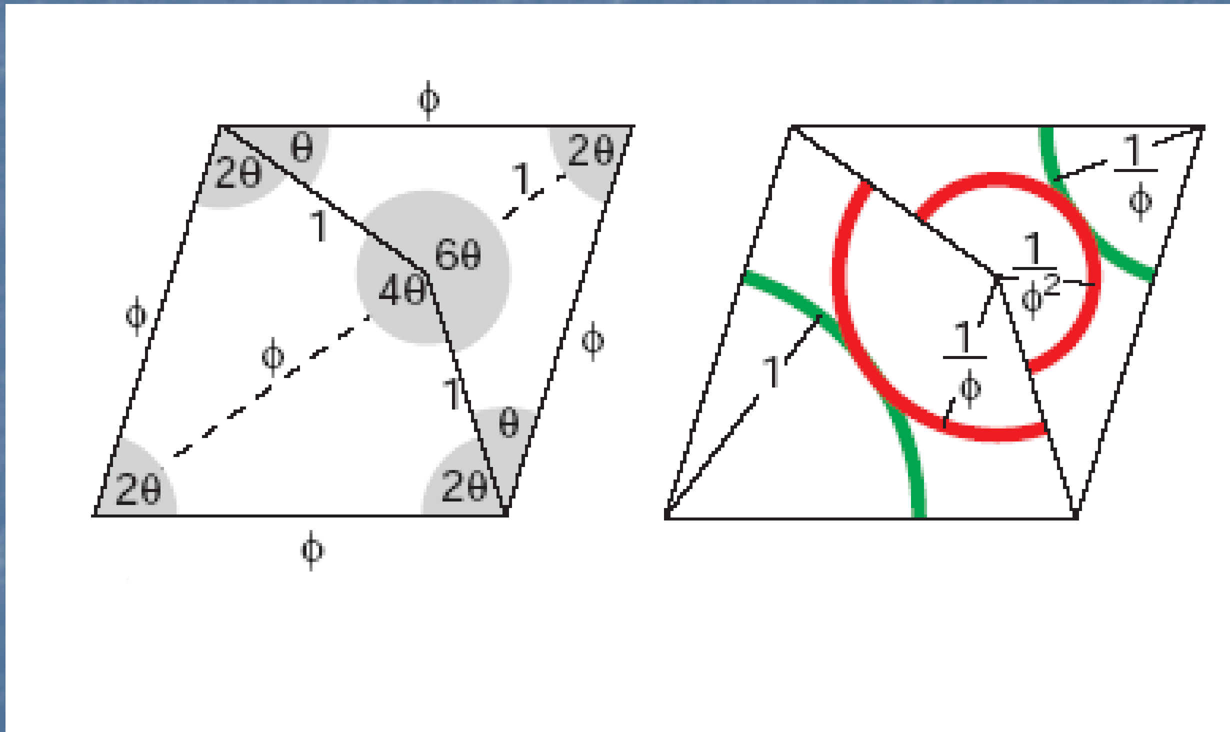
=> A structure but not boring

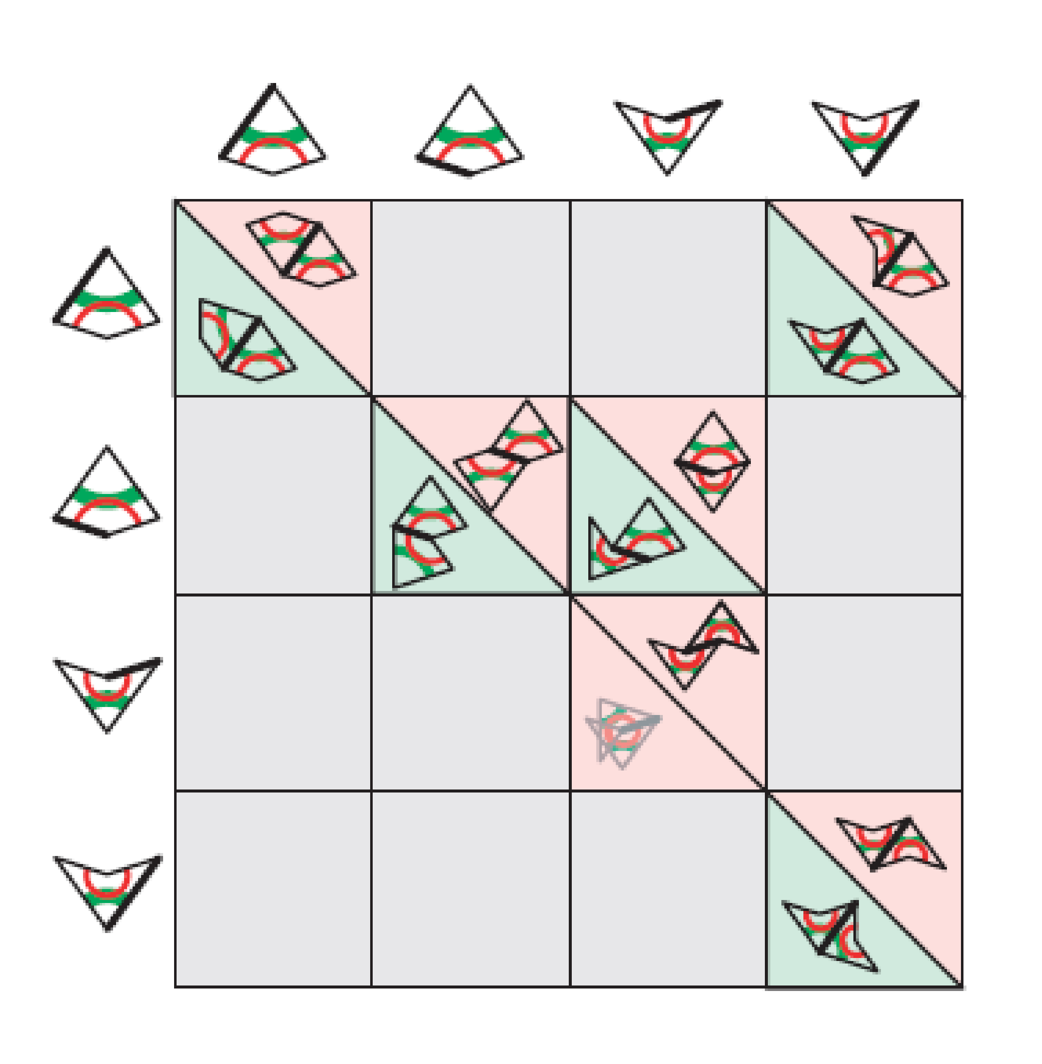
Similarly matrices for geometry, materials, tekoucí lávu etc. – a structure but not a repetition

How to decorate

- Patterns either symmetrical according to the drawn axis or closed "inside"

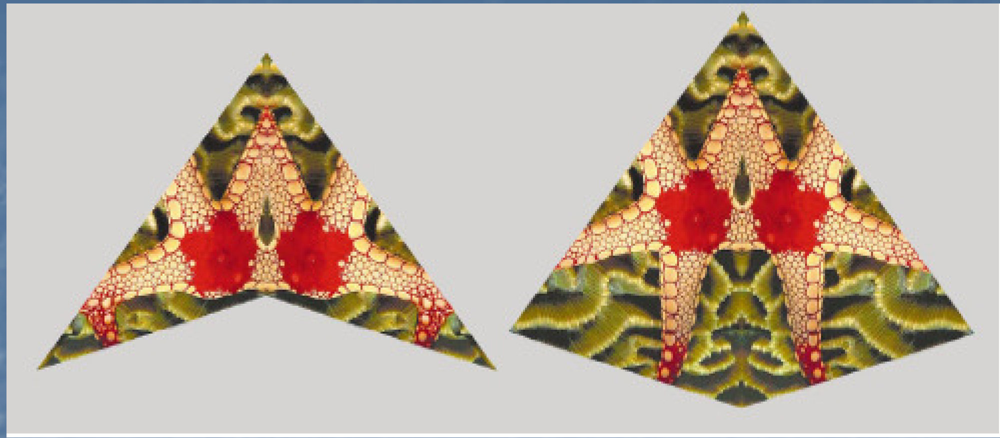


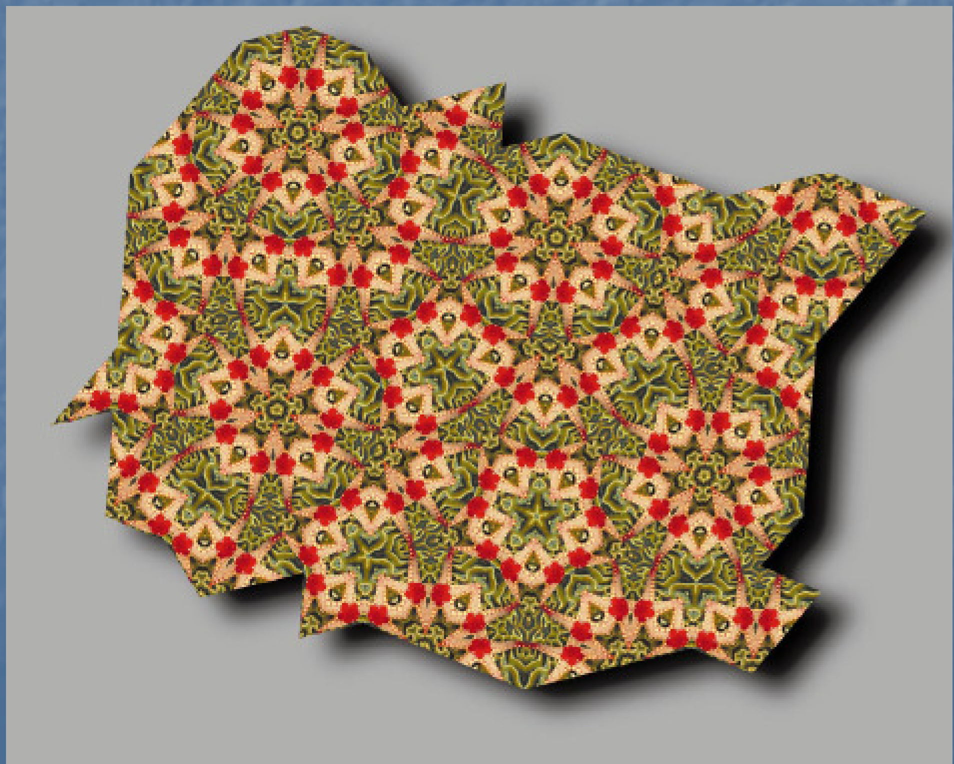




- Pink – incorrect,
- Green – correct
- Thick line – the edge of connection
- The lower left corner of the matrix the same as the upper right
- Grey squares – edges of different length, do not fit

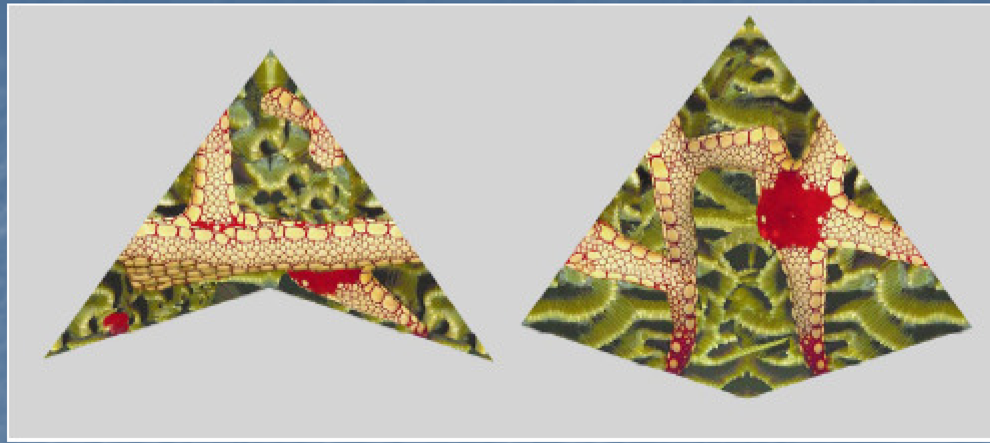
Examples
of decorations



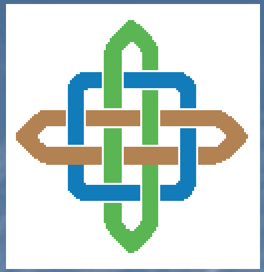


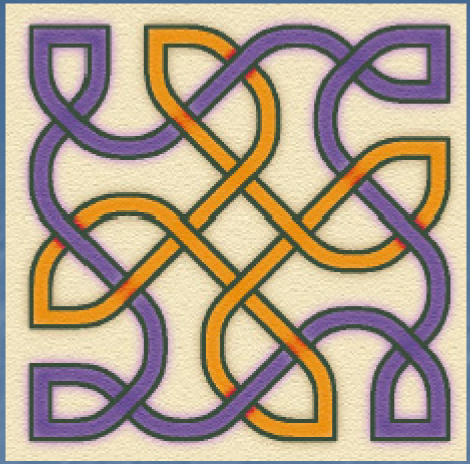
Examples
of decorations

With some asymmetry



3. Celtic knotwork

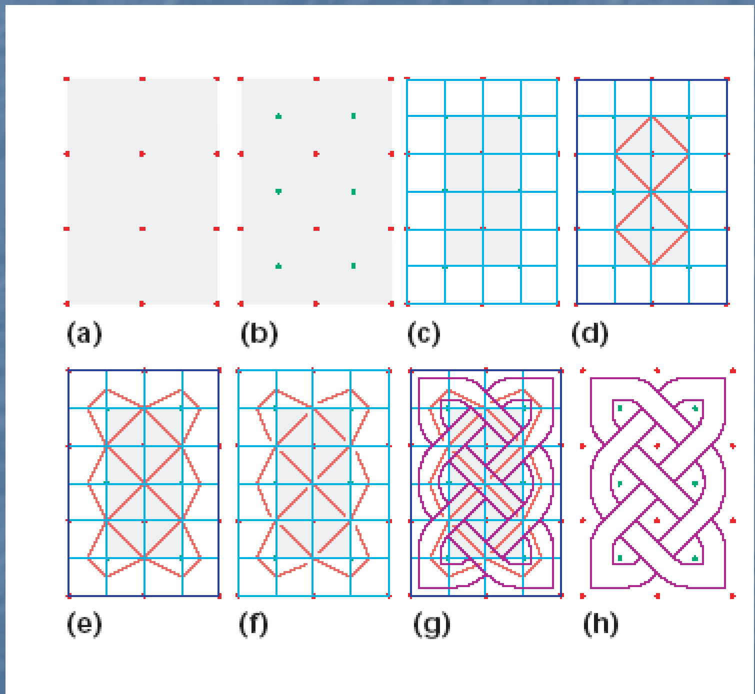




- From about the 6^{th} century, a decoration of religious texts at the Irish monks
- G. Bain in 1951 – proposed a simple construction algorithm on the basis of study of old celtic manuscripts
- Algorithm: based on a grid – from a fundamental regular pattern
- Classical celtic knotwork usually one thread/strip but more can me done

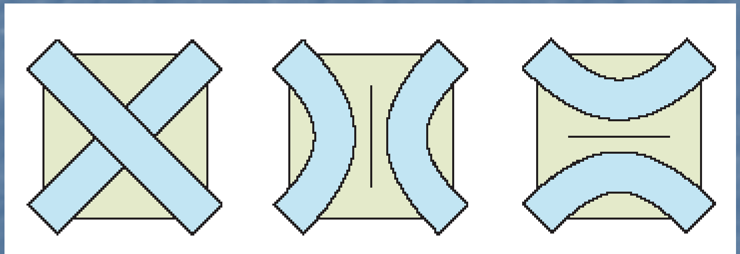
Basic steps

- Ex.: a pattern 2x3



a) Primary grid 2x3 squqres
b) A new point to the centre of each square - a secundary grid
c) Tertiary grid
d) Basic pattern added
e) Outer connection added
f) The same with an enlargement of mutual overlap – the first step is chosen, then it is given
g) A strip around the skeleton added
h) Result incl. the original grid

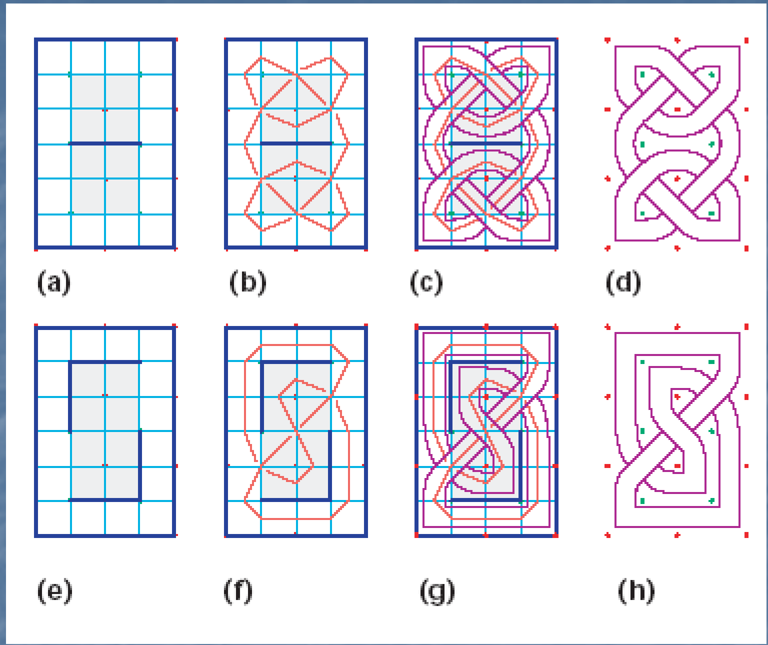
- In this way any resolution
- If vertical and horizontal sizes have no common divisor, then 1 strip, else several strips
- Enrichment: breaklines – they redirect the intersection



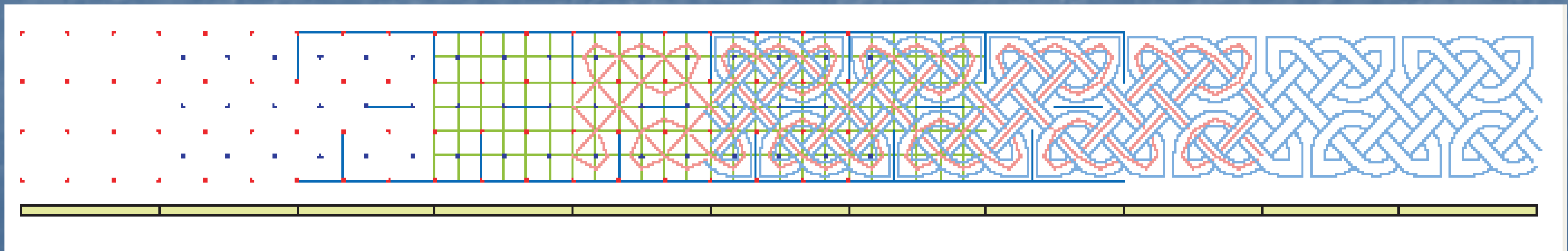
- Breaklines won't intersect, they can join horizontal or vertical neighbours in the same primary or secondary grid (not primary with secondary, not distant neighbours ...)

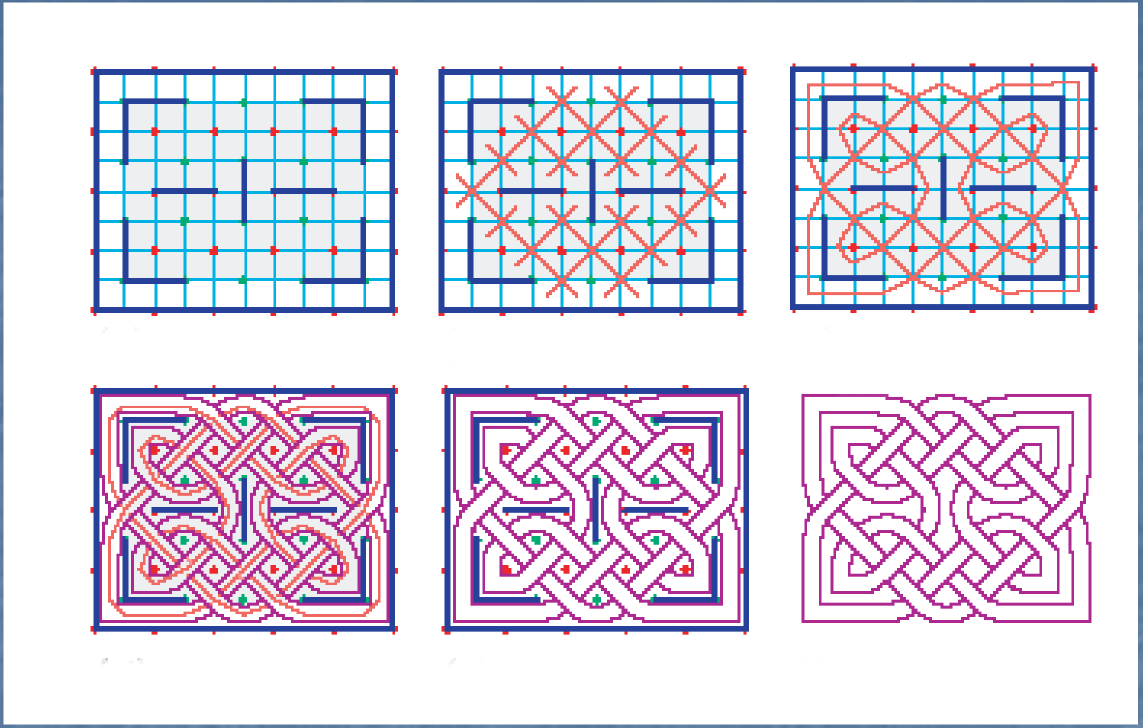
a) Breakline in primary grid
b) Resulting skeleton
c) Strip
d) Result

e) 4 breaklines
f) Skeleton
g) Strip
h) Result



Example of a strip construction:



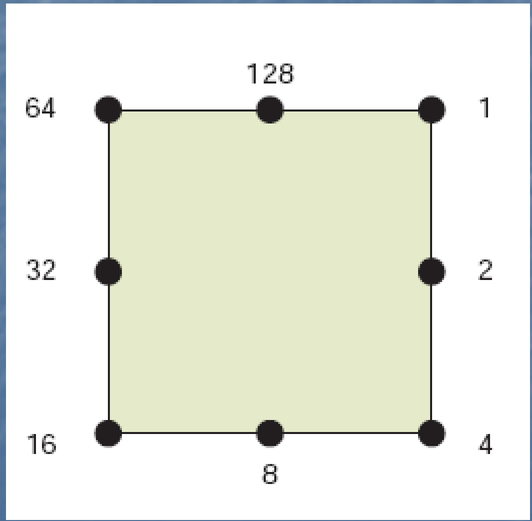


More advanced example

- The breakline may change not only the strip type but also the number of strips
- Primary grid $x*y$ cells can have 1 to xy strips

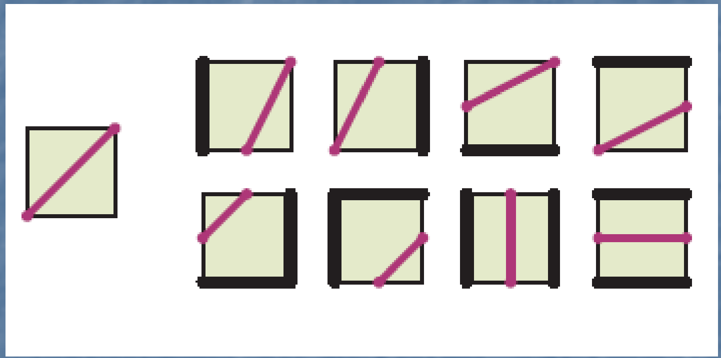
Implementation

- Primary grid xy => a data structure 2x2y for tertiary grid
- Information for one cell:
  - Break identification – whether the upper left corner of the cell has a break, has a line to the right, to the bottom or both
  - a Visited flag
  - No of the strip
  - Edge code – where the line touches the cell - LOR of codes, e.g., a line from lower left corner to the centre on the right – code 18



Skeleton drawing 1

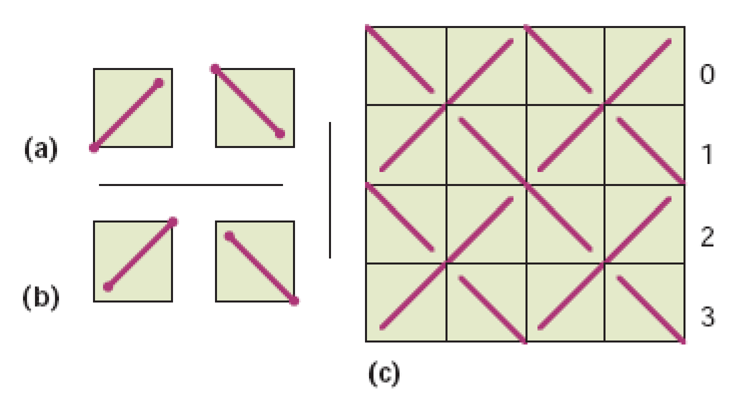
- Go through all tertiary cells, set edge codes
- Without break: the skeleton in the cell in the upper left corner goes from lower left to upper right corner => check of the breaklines for this cell and corresponding modification of the diagonal
- max. 2 breaks per cell (<= tertiary cell)
- After the cell is coded, move right, here the opposite direction of the diagonal, etc.



Skeleton drawing 2

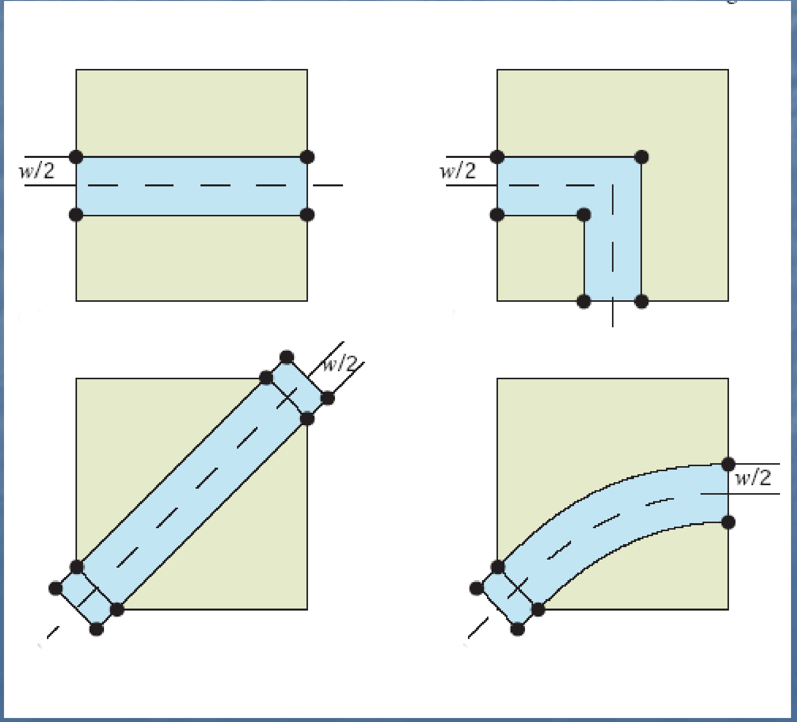
- After all the cells have been evaluated, we set the number of strip in the first cell and each visited cell gets Visited <= true (only 1 stip in each cell)
- Continue according to the skeleton into further cells
- When the strip is being closed, look for another unvisited cell, it gets one higher number of strip, etc.
- Drawing: inspect cells in the order of strip, alternate the strip drawing 'above' and 'below'

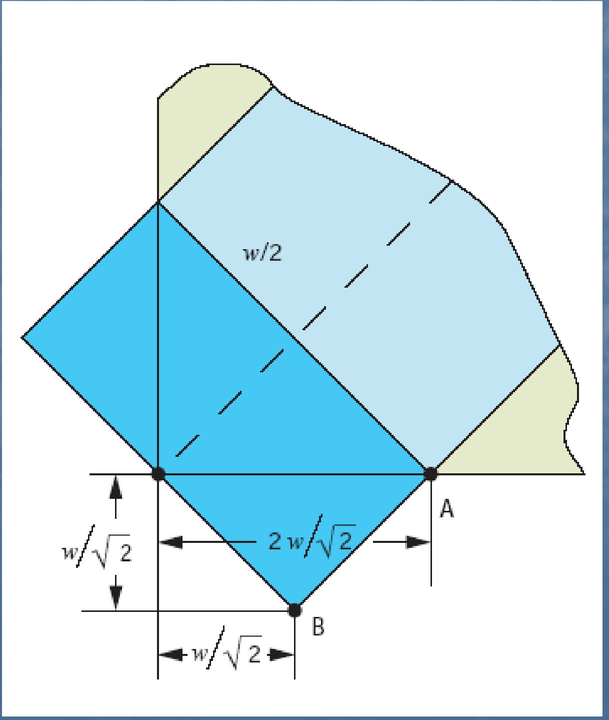
a) Even rows
b) Odd rows
c) Together



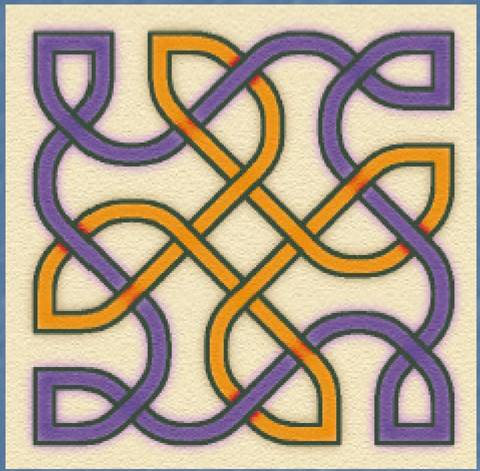
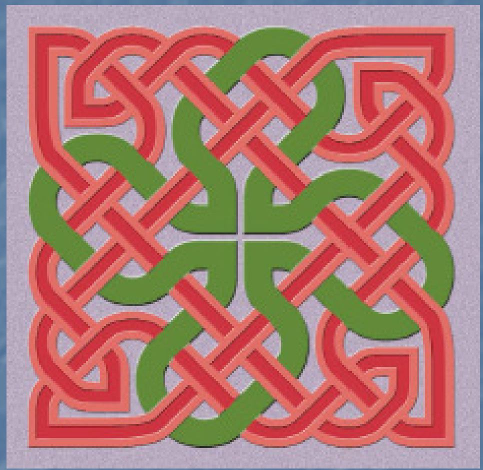
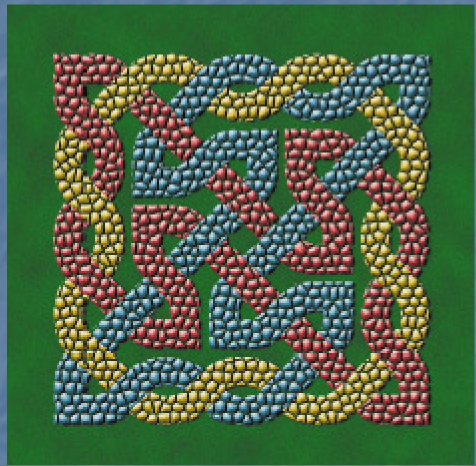
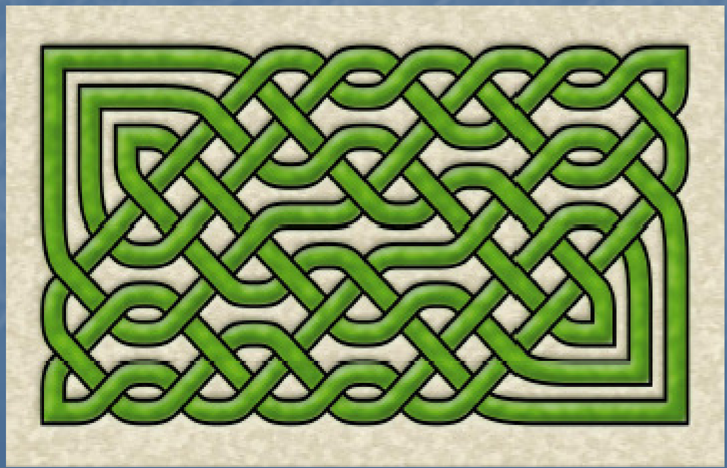
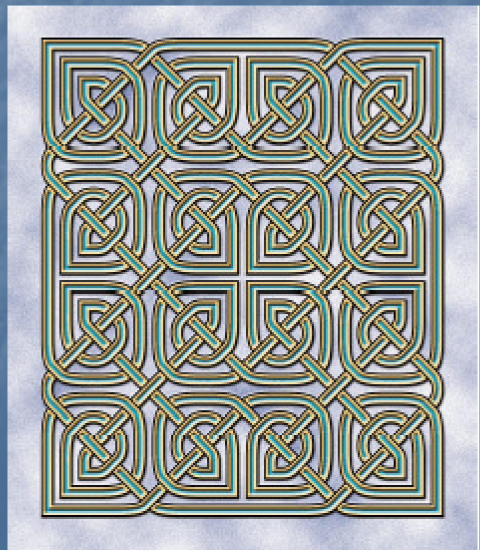
Strip drawing

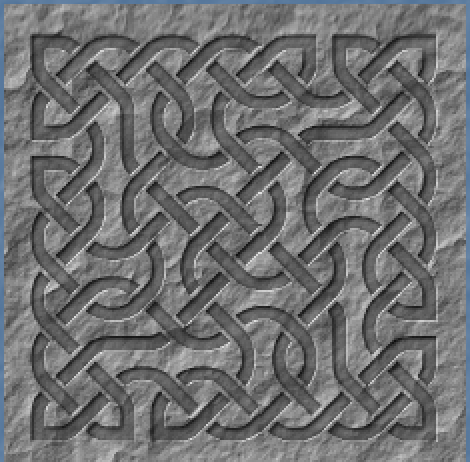
- Instead of the skeleton, draw a strip of the width *w*
- 6 elements – short, long arc and 4 below, parameters are the strip width and orientation

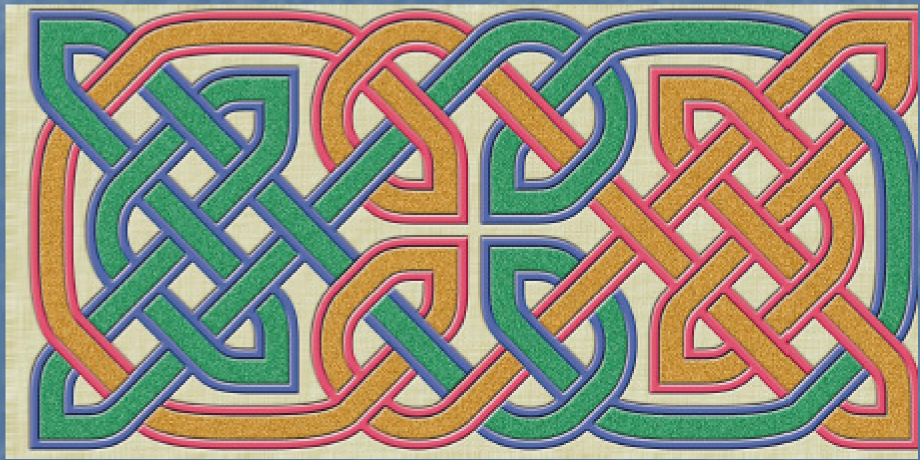




- Results can be further improved in Photoshop

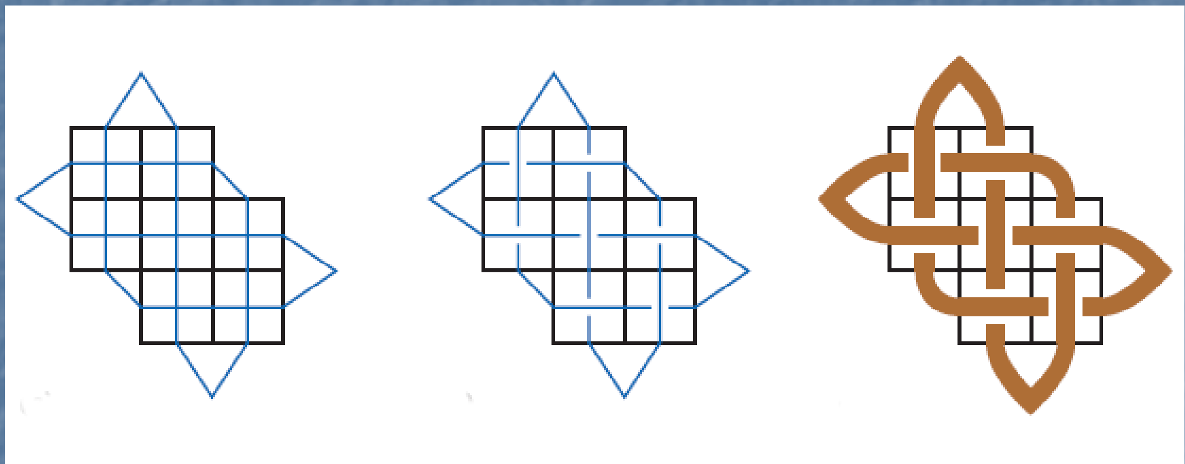






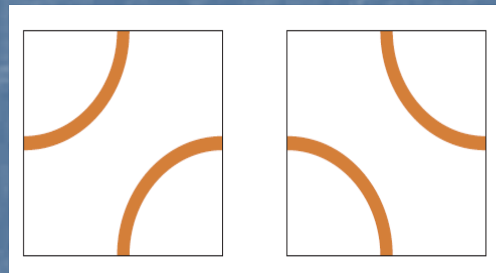




Modification 1

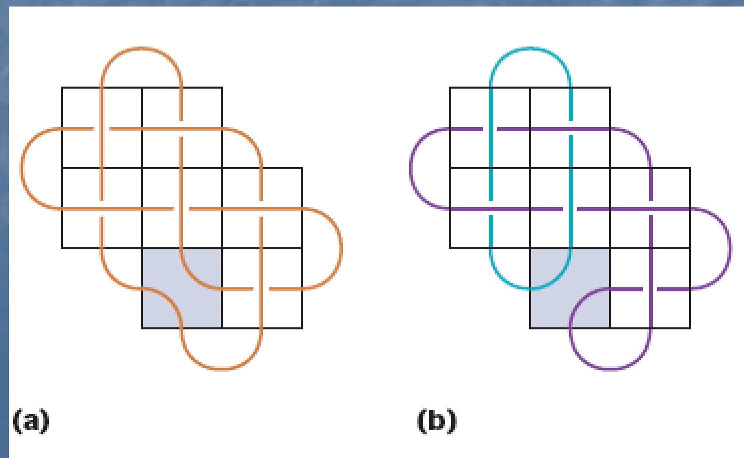
- Instead of intersection, connect the opposite sides of the cell





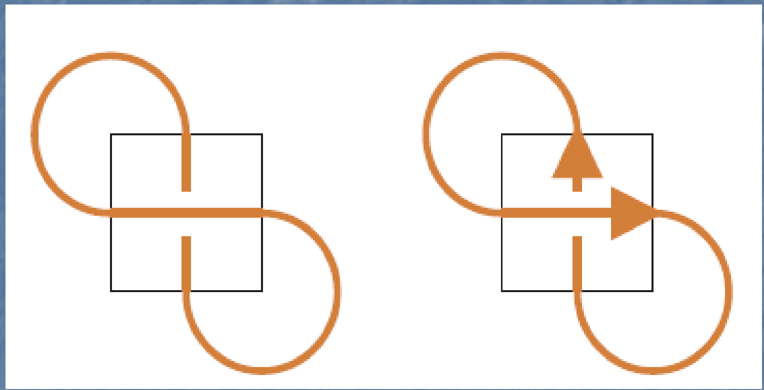
=> By intersection removal, number of strips may change by +1/-1

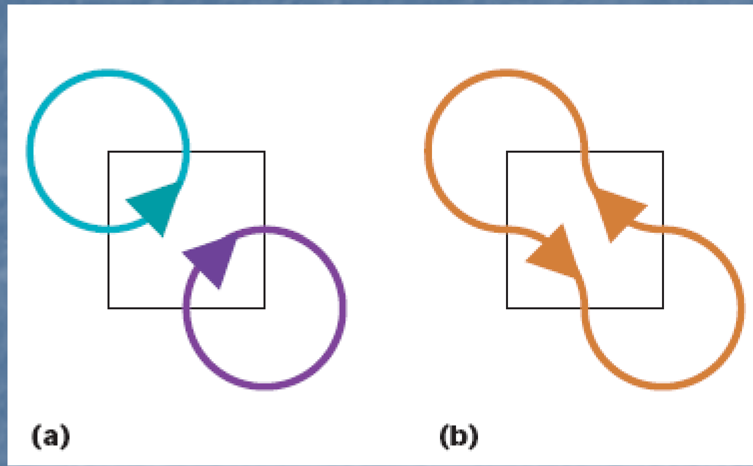
a) 1 strip
b) 2 strips



Modification 2

- Add the orientation: either as a new pattern or to find whether the number of strips was preserved or not

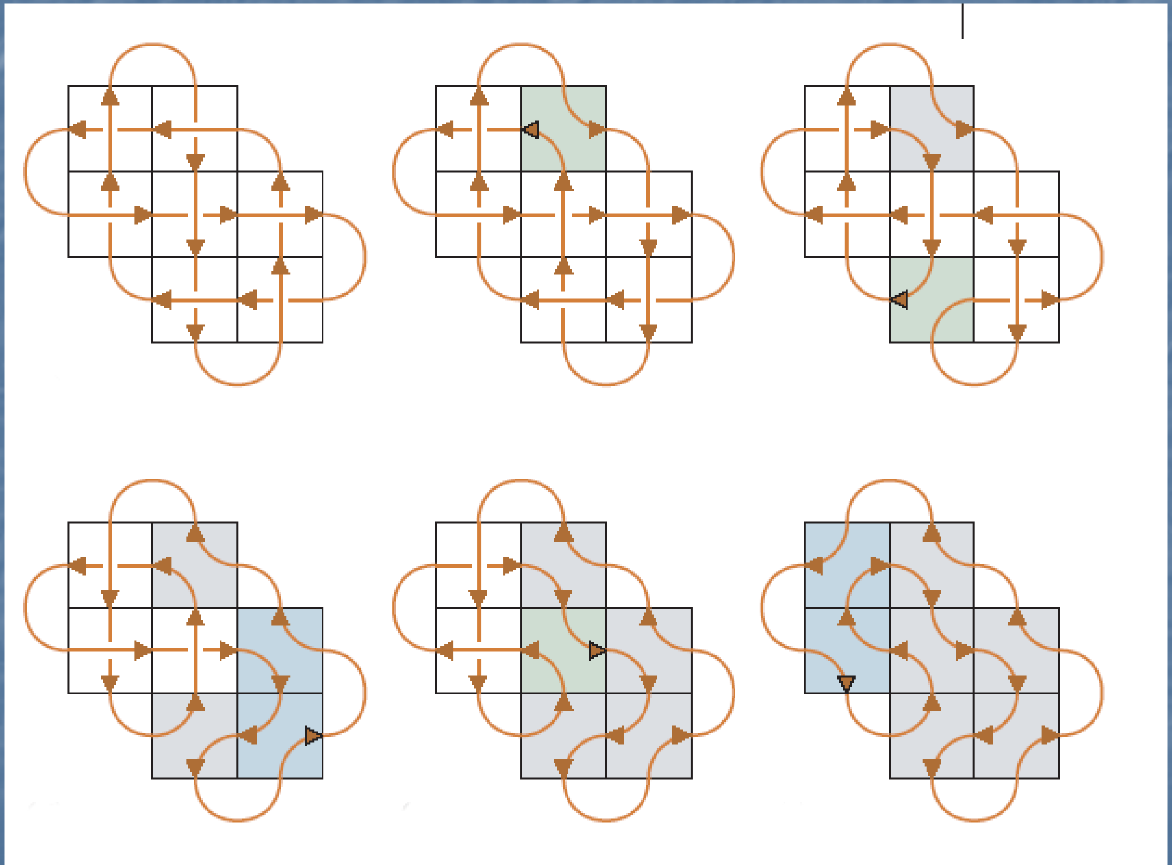




a) 1 more strip
b) No change

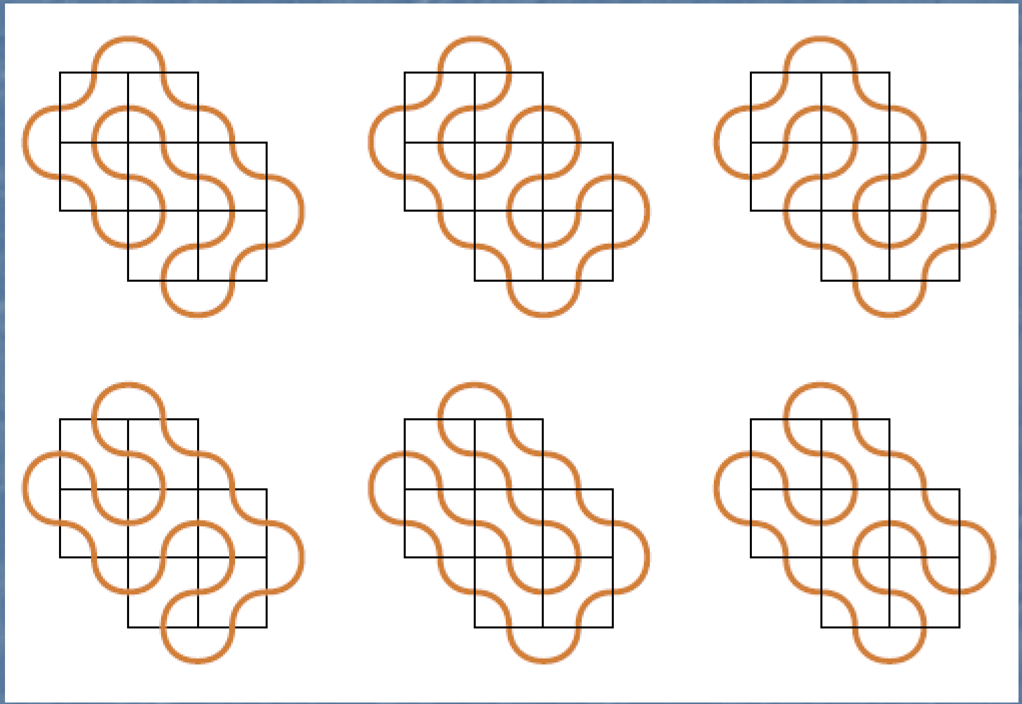
Modification 3 - snakes

- Snakes – by removal of all intersections
- The shape according to the order of changes



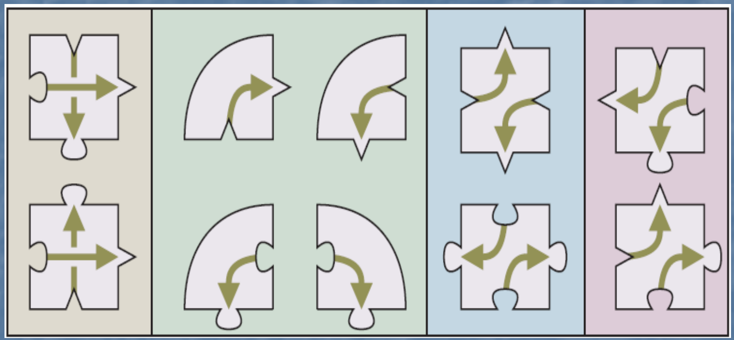
Orientation must be changed during the construction

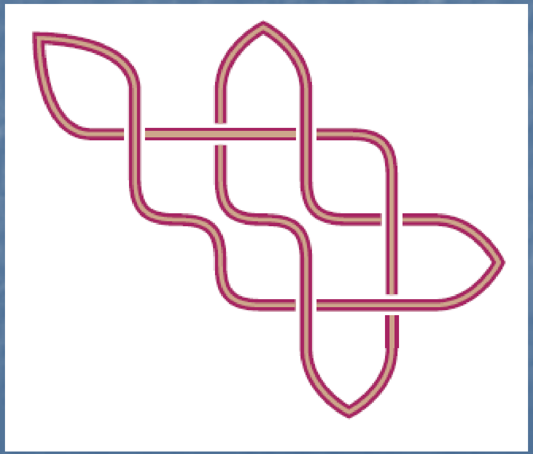
6 versions of snakes obtained from the running example

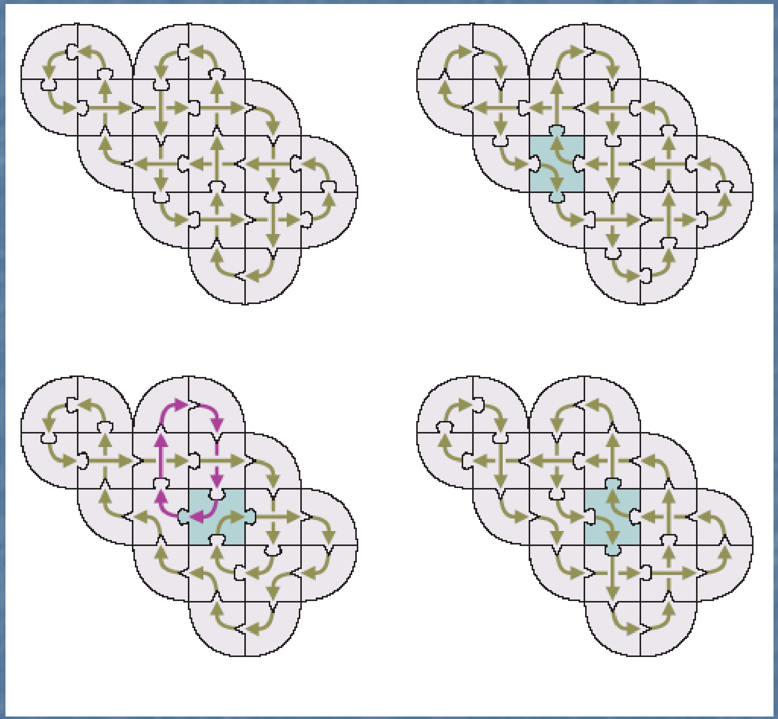


Modification 4

- Can be done as a tile, see before

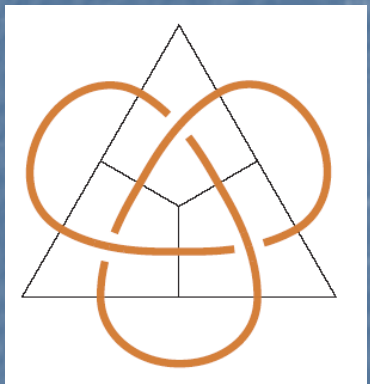


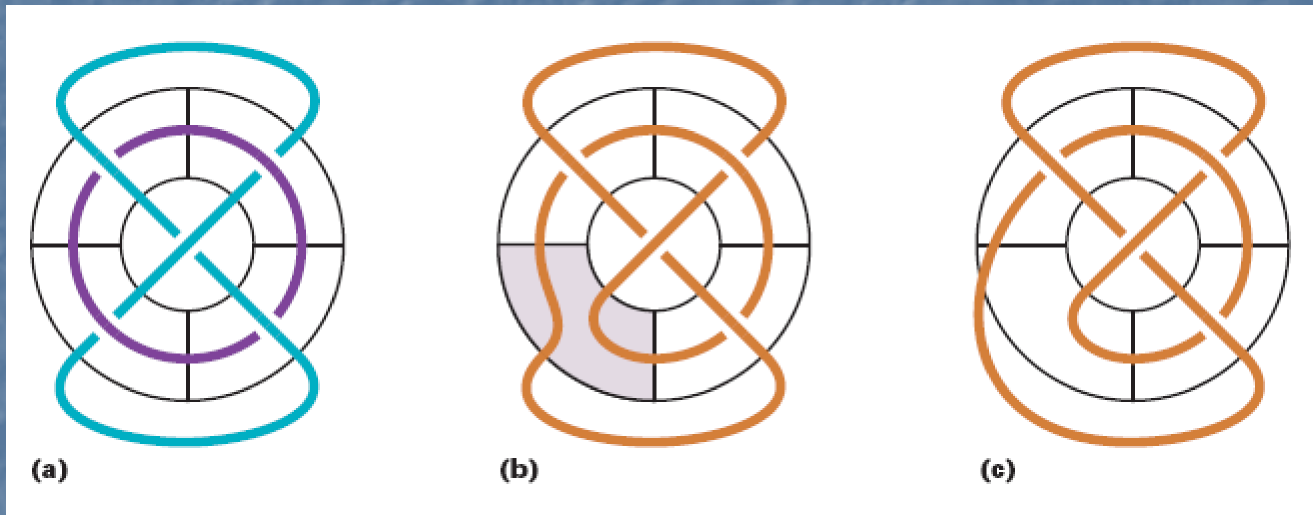




Modification 5

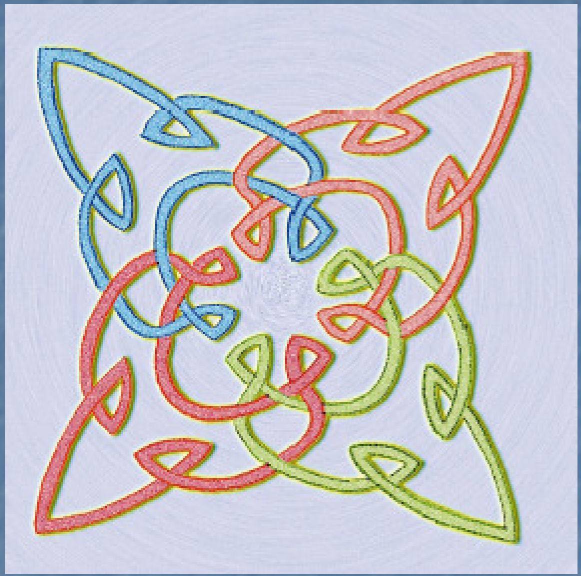
- Instead of 4gons, a matrix of triangles, circular slices – still 4 sides, but curved and different connectivity

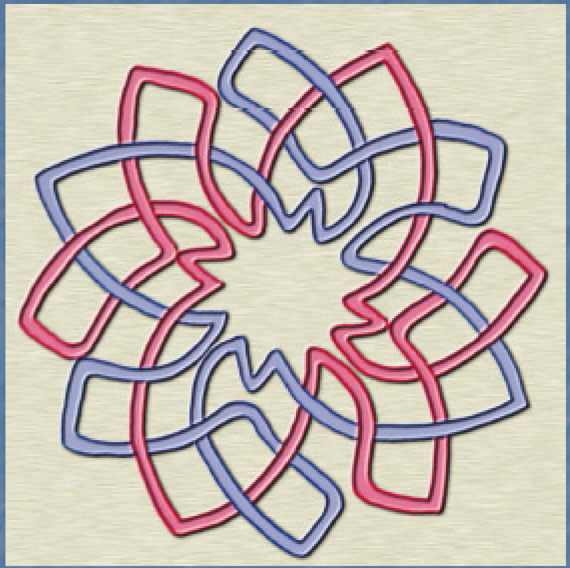


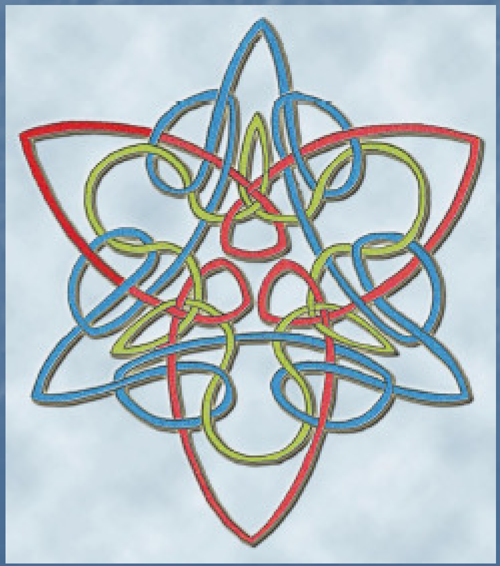


a) 2 strips
b) After intersection replacement
c) b) smoothed

Examples (improved in Photoshop)

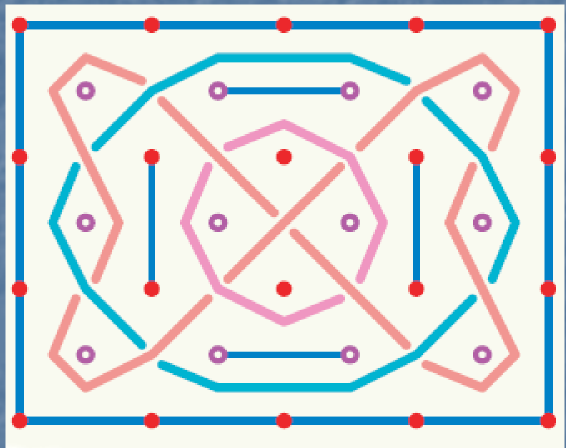


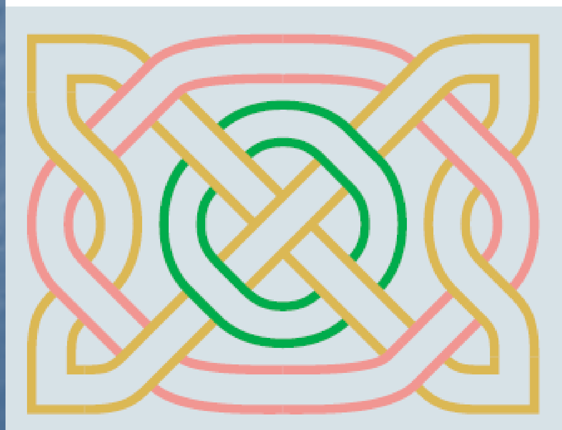




Modification 6

- 3D version





Strip lift into 3D

- A cubic curve *s(x)* for *x* from <0,1>, x modified by another curve *t*, which using an optional parameter *n* makes various types of the lift

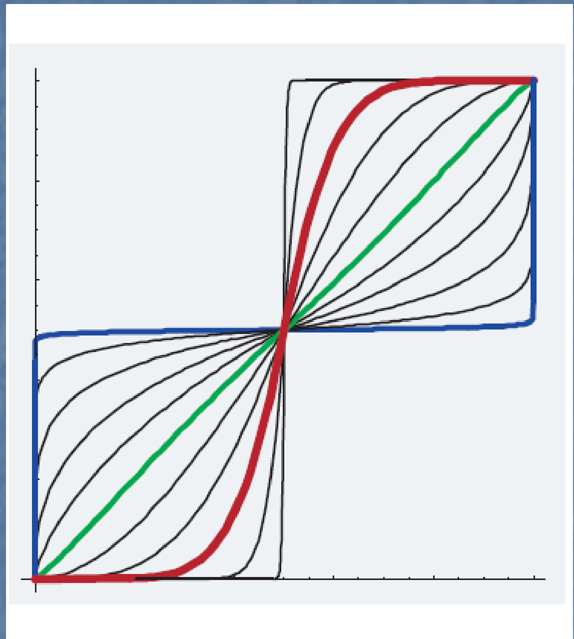
$$s(x) = -2x^3 + 3x^2$$

$$t(x,n) = s((2x)^n / 2)$$

$$u(x,n) = \text{if } x < 0.5 \text{ then } t(x,n) \quad \text{else } 1 - t(1-x,n)$$

$$x \in \langle 0,1 \rangle$$

Strip lift into 3D 2



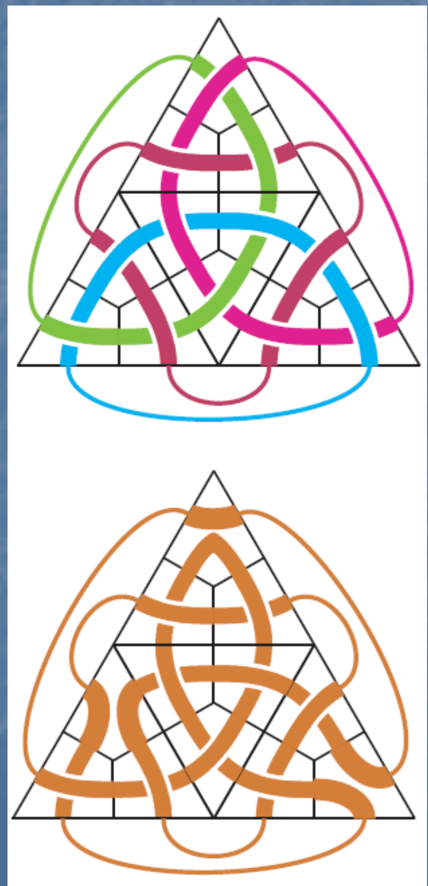
u(x,n) for various *n*

(*n*=0.005 blue,
n=0.6 green,
n=3.5 red
- value for the previous page)

A cylinder instead of a strip: Bézier curve for the strip axis, used in 3D Max as a path for an extruded spline or a lofted surface

Modification 7

- Other than rectangular boxes: possible, but more difficult – usually there are not 4 sides



Tetrahedron faces